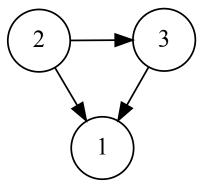
CS41 Lab 5

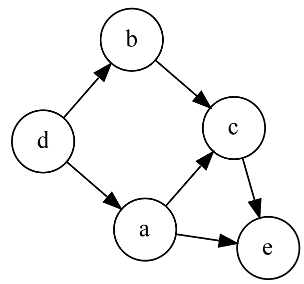
September 28, 2021

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students. You will not be handing in solutions; the primary purpose of these labs is to have a low-pressure space to discuss algorithm design. However, it will be common to have some overlap between lab exercises and homework sets. The goal of this lab session is to gain more experience with algorithm design, particularly using directed graphs.

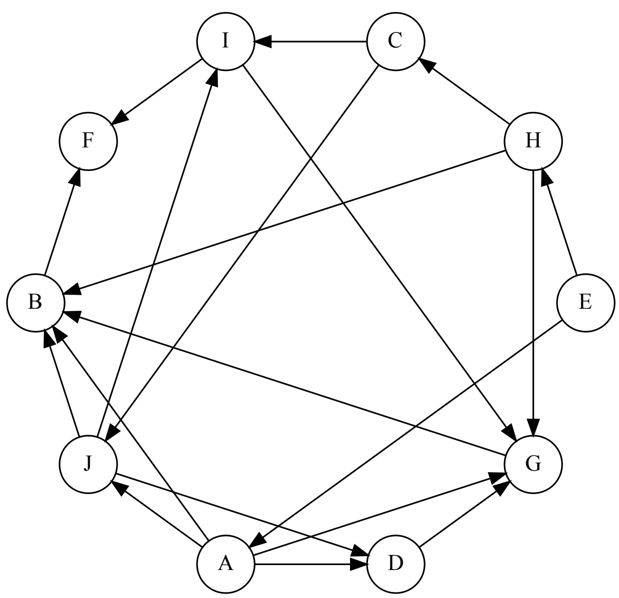
1. **Topological Sorting.** For each of the following directed graphs, determine whether or not the graph is *acyclic*. If the graph is cyclic, identify a cycle. If the graph is acyclic, give a topological ordering of vertices.



Directed graph G_1.



Directed graph G_2.



Directed graph G_3.

2. **Butterfly Classification.** (K& T 3.4) Some of your friends are *lepidopterists* — they study butterflies. Part of their recent work involves collecting butterfly specimens and identifying what species they belong to. Unfortunately, determining distinct species can be difficult because many species look very similar to one another.

   During their last field expedition, your friends collected n butterfly specimens and believe the specimens come from one of two butterfly species (call them species A and B.) They'd like to divide the n specimens into two groups—those that belong to A and those that belong to B. However, it is very hard for them to directly label any one specimen. Instead, they adopt the following approach:

   For each pair of specimens i and j, they study them carefully side by side. If they're confident enough in their judgement, they will label the pair as *same* (meaning they are confident that both specimens belong to the same species) of *different* (meaning they believe that the specimens belong to different species). If they are not confident, they do not label the specimens. Call this labeling (either (i, j) are the same or (i, j) are different) a *judgement*.

   A set of judgements is **consistent** if it is possible to label each specimen either A or B in such a away that for each pair (i, j) labeled "same", it is the case that i and j have the same label, and for each pair (i, j) labeled "different", it is the case that i and j have different labels.

   Design and analyze an algorithm which takes n butterfly specimens and m judgements, and outputs whether or not the judgements are consistent. Your algorithm should run in $O(n+m)$ time.

3. **Claire's Coffee Corner.** Claire has a coffee shop, famous for its elaborate special-order coffees. Each beverage costs $2, no matter what unusual ingredients are used or how long it takes to prepare the drink. Needless to say, this deal attracts a lot of regular customers, some of whom order very unusual concoctions that take a long time to prepare.

   Claire's regular customers are impatient and demanding. They want coffees made *exactly* according to their special order, and if their drinks aren't started as soon as they enter the shop, they get angry and storm out.

   Claire used to have many employees to help her make the coffees, but unfortunately, all of Claire's coffee clerks went on strike, leaving Claire alone to make the coffees. Claire wants to make as many coffees as possible, but now can only prepare one drink at a time. Coffee preparation requires a lot of attention, so once Claire begins to make a drink, she must focus on that drink until it is ready. Only after it is prepared can Claire begin the next drink.

   Design and analyze a strategy to help Claire quickly decide which drinks to make. Your strategy should take as input the information about regular customers (when they arrive and how long their drink takes to make) and output which drinks Claire should make. Does your strategy guarantee that Claire prepares the maximum number of coffees? If so, say why. Otherwise, give an example when it does not maximize the number of coffees prepared.