CS41 Lab 3

September 14, 2021

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students. You will not be handing in solutions; the primary purpose of these labs is to have a low-pressure space to discuss algorithm design. However, it will be common to have some overlap between lab exercises and homework sets.

The goal of this lab session is to gain more experience with asymptotic analysis. Do not expect to complete all parts of all problems by the end of the lab. Consider it a successful lab session if you can complete the first two problems and make reasonable progress on either the third or fourth problem. The fifth problem is icing on the cake, and very open-ended, so you should consider it last.

For these problems, your example functions should have domain and range the positive integers $\mathbb{N}$. Keep it simple.

1. **Rates of Growth.** Arrange the following functions in ascending order of growth rate. That is, if g follows f in your list, then it should be the case that f is $O(g)$.

   - $f_1(n) = n^{2.5}$
   - $f_2(n) = \sqrt{2n}$
   - $f_3(n) = n + 10$
   - $f_4(n) = 10^n$
   - $f_5(n) = 100^n$
   - $f_6(n) = \log_{1.1}(n)\sqrt[3]{n}$
   - $f_7(n) = n^n$
   - $f_8(n) = n^2\log_2(n)$
   - $f_9(n) = n^{\log_2(n)}$

   No proofs are necessary; just arrange the functions in ascending order of growth.

2. **Big-O facts.** Assume you have functions f, g, and h from $\mathbb{N}$ to $\mathbb{R}^{\geq 0}$. Prove the following:

   (a) If f is $O(h)$ and g is $O(h)$, then the function $f + g$ is $O(h)$.

   (b) If f is $O(g)$ and g is $O(h)$, then f is $O(h)$.

3. **Asymptotic analysis.** Assume you have functions f and g such that $f(n)$ is $O(g(n))$. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.

   (a) $\log_2(f(n))$ is $O(\log_2(g(n)))$.

   (b) $2^{f(n)}$ is $O(2^{g(n)})$.

   (c) $(f(n))^2$ is $O((g(n))^2)$.

4. **Asymptotic Proofs.** Let $f(n) = 2(\log(n))^3 + 6$ and $g(n) = 5n^{1/4} + 10$. Prove that $f(n) = O(g(n))$. You may use techniques and facts from class and the textbook. Your proof should be complete and formal.

5. **The Wedding Planner Problem.** Imagine you are a wedding planner. Among other tasks, you must help your customers with their guest lists. Couples come to you with lists of people they might invite to their wedding. They also come with demands – Alice and Bob need to be invited, but if Bob is invited, do not invite Carol (they have history). When Carol, Dave,

and Eve get together, they only talk about Beyoncé (their favorite musician), so don't invite all three. However, any two of them is ok. Call these conditions *constraints*.

People at weddings are **demanding**. Everything needs to be exactly perfect, and they will blame you if even one constraint is not satisfied. You're not even sure if this is possible, and it when it's not, it would be nice to have a convincing argument for the wedding couple.

Design an algorithm that takes a list of people, and a list of constraints, and outputs YES if there is an invite list that satisfies every constraint. Otherwise, output NO.