CS41 Lab 3

September 16, 2019

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students. You will not be handing in solutions; the primary purpose of these labs is to have a low-pressure space to discuss algorithm design. However, it will be common to have some overlap between lab exercises and homework sets.

The goal of this lab session is to gain more experience with asymptotic analysis. Do not expect to complete all parts of all problems by the end of the lab. Consider it a successful lab session if you can complete the first problem and make reasonable progress on either the second or third problem.

For these problems, your example functions should have domain and range the positive integers $\mathbb{N}$.

1. **Rates of Growth.** Arrange the following functions in ascending order of growth rate. That is, if g follows f in your list, then it should be the case that $f = O(g)$.

   - $f_1(n) = n^{2.5}$
   - $f_2(n) = \sqrt{2n}$
   - $f_3(n) = n + 10$
   - $f_4(n) = 10^n$
   - $f_5(n) = 100^n$
   - $f_6(n) = \log_{1.1}(n)\sqrt[3]{n}$
   - $f_7(n) = n^n$
   - $f_8(n) = n^2 \log_2(n)$
   - $f_9(n) = n^{\log_2(n)}$

   No proofs are necessary; just arrange the functions in ascending order of growth.

2. **Asymptotic Analysis.** Assume you have functions f, g, and h. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.

   (a) If f is $\Omega(g)$ and g is $\Omega(h)$, then f is $\Omega(h)$.

   (b) If f is $O(h)$ and g is $O(h)$, then $f + g$ is $O(h)$.

   (c) If f is $O(h)$ and g is $O(h)$, then $f \cdot g$ is $O(h)$.

   (d) If f is not $O(g)$, then g is $O(f)$.

3. **Asymptotic Proofs.** Let $f(n) = 2(\log(n))^3 + 6$ and $g(n) = 5n^{1/4} + 10$. Prove that $f(n) = O(g(n))$. You may use techniques and facts from class and the textbook. Your proof should be complete and formal.