CS41 Lab 2

September 9, 2019

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students. You will not be handing in solutions; the primary purpose of these labs is to have a low-pressure space to discuss algorithm design. However, it will be common to have some overlap between lab exercises and homework sets.

Note: If you do not feel fully confident when it comes to proof using induction, I strongly encourage you to focus on the initial problem first.

1. **Induction.** Using induction, show that the following summations hold for all $n \geq 0$.
   - $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$.
   - $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$.
   - for all positive $c \neq 1$, $\sum_{k=0}^{n} c^k = \frac{c^{n+1} - 1}{c - 1}$.
2. **Asymptotic analysis.** Assume you have functions f and g such that $f(n)$ is $O(g(n))$. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.
   (a) $\log_2(f(n))$ is $O(\log_2(g(n)))$.
   (b) $2^{f(n)}$ is $O(2^{g(n)})$.
   (c) $(f(n))^2$ is $O((g(n))^2)$.
   (d) If $g(n)$ is $O(h(n))$, then $f(n)$ is $O(h(n))$.
   (e) $g(n)$ is $\Omega(f(n))$.