This week, we’ve been discussing ways to classify problems according to their difficulty (“computa-
tional complexity”), using the notions of polynomial-time reduction, polynomial-time verifier, and
NP-completeness. In this lab, we’ll look at two NP-COMPLETE problems.

   Recall that to show a problem $A \in$ NP-COMPLETE, it suffices to:
   
   - Prove that $A \in$ NP.
   - Choose a problem $B$ known to be NP-COMPLETE.
   - Reduce $B \leq_p A$.

During this lab, focus initially on the reductions, and not the formal proofs.

1. Show that $3$-Sat $\in$ NP-COMPLETE, by reducing from Sat. Given an instance $X$ of Sat (i.e.,
a list of $n$ variables and $m$ clauses), you should create an instance $Y$ of 3-Sat (i.e., a list of $n'$
variables and $m'$ clauses, each clause having three literals) such that $Y \in$ 3-Sat iff $X \in$ Sat.

2. In the third exercise, you will show that Three-Coloring is NP-COMPLETE. Before getting
there, it will be helpful to create some interesting three-colorable graphs. In all of the following
exercises, you are to create a three-colorable graph (say the colors are red, blue, green) with
certain special properties. The graphs you create should include three vertices marked $a, b, c$
but can (and often will) include other vertices. Except for the properties specified, these
vertices should be unconstrained. For example, unless the problem states that e.g. $a$ cannot
be red, it must be possible to color the graph in such a way that $a$ is red. (You may fix colors
for other vertices, just not $a, b, c$, and not in a way that constrains the colors of $a, b, c$.)

   (a) Create a graph such that $a, b, c$ all have different colors.
   (b) Create a graph such that $a, b, c$ all have the same color.
   (c) Create a graph such that $a, b, c$ do NOT all have the same color.
   (d) Create a graph such that none of $a, b, c$ can be green.
   (e) Create a graph such that none of $a, b, c$ are green, and they cannot all be blue.

3. Show that Three-Coloring $\in$ NP-COMPLETE.
   (Hints: reduce from 3-Sat. Associate the color red with TRUE and the color blue with FALSE.)