CS41 Lab 3

The lab and homework this week center on graph algorithms for undirected graphs. The following definitions might be helpful/relevant.

- A *path* P on a graph $G = (V, E)$ is a sequence of vertices $P = (v_1, v_2, \ldots, v_k)$ such that $(v_i, v_{i+1}) \in E$ for all $1 \leq i < k$.
- A path is *simple* if all vertices are distinct.
- The *length* of a path $P = (v_1, \ldots, v_k)$ equals $k - 1$. (Think of the path length as the number of edges needed to get from v_1 to v_k on this path).
- A *cycle* is a sequence of vertices $(v_1, \ldots, v_k)$ such that $v_1, \ldots, v_{k-1}$ are all distinct and $v_k = v_1$. A cycle is odd (even) if it contains an odd (even) number of edges.

1. **Cycle Detection.** Design and analyze an efficient algorithm for finding a cycle in a graph. Your algorithm should take as input a graph $G = (V, E)$ and report a cycle (or output NO if no cycle exists). If there are multiple cycles in the graph, your algorithm should just output one.

2. **Testing Bipartiteness.** Call a graph $G = (V, E)$ **bipartite** if you can partition V into sets A and B such that all edges $e \in E$ have one vertex in A, one in B. Design an analyze an algorithm to test a graph for bipartiteness.

   Hint: An alternate definition is that $G = (V, E)$ is bipartite if you can color vertices in V by one of two colors so that each edge is **bichromatic**: for any $\{u, v\} \in E$, vertex u is a different color from vertex v.

3. **Testing Tripartiteness.** Call a graph $G = (V, E)$ *tripartite* if V can be partitioned into disjoint sets A, B, C such that for any edge $(u, v) \in E$, the vertices u, v lie in different sets. Design and analyze an algorithm to test a graph for tripartiteness.