Algorithm 3 analysis

```
ALGORITHM3()
k = 1.
while you haven't found your friend:
    walk k miles east
    return to start
    walk k miles west
    return to start
    k = 2 * k.
```

Now let's analyze this algorithm.

Correctness: Clearly you do eventually reach your friend, and then you stop, so the algorithm is correct.

Efficiency: Suppose your friend starts m miles from your starting location. How far do you have to walk, following this algorithm, before you find her?

In one complete iteration of the algorithm, you walk $4k$ miles. How many complete iterations do you do?

Let's say that we stop on the t^{th} iteration, so $2^{t-1} < m \leq 2^t$. (The first inequality is because we finished iteration $t-1$ and still had not finished; the second inequality is because we find her on this iteration.)

- Case 1: Your friend is m miles east of you.
  The total distance walked

$$
\begin{aligned}
&= 4 \cdot 1 + 4 \cdot 2 + 4 \cdot 4 + 4 \cdot 8 + \ldots + 4 \cdot 2^{t-1} + m \quad \text{(the last, partial iteration adds the } +m \text{ term)} \\
&= \sum_{k=0}^{t-1} \left(4 \cdot 2^k\right) + m \\
&= 4 \cdot \sum_{k=0}^{t-1} \left(2^k\right) + m \\
&= 4\left(2^t - 1\right) + m && \left(\text{by the fact that } \sum_{k=0}^{t-1} 2^k = 2^t - 1\right) \\
&< 4\left(2m - 1\right) + m && \left(\text{because } 2^{t-1} < m \text{ so } 2^t < m\right) \\
&= 8m - 4 + m \\
&= O(m)
\end{aligned}
$$

- Case 2: Your friend is m miles west of you.

Total distance walked

$$
\begin{aligned}
&= 4 \cdot 1 + 4 \cdot 2 + \ldots + 4 \cdot 2^{t-1} + \left(2^t + 2^t + m\right) && \text{(don't forget the partial last iteration!)} \\
&= \sum_{k=0}^{t-1} \left(4 \cdot 2^k\right) + 2^{t+1} + m \\
&= 4 \cdot \sum_{k=0}^{t-1} \left(2^k\right) + 2^{t+1} + m \\
&= 4\left(2^t - 1\right) + 2^{t+1} + m && \text{(by the fact that } \sum_{k=0}^{t-1} 2^k = 2^t - 1\text{)} \\
&< 4\,(2m - 1) + 4m + m && \text{(because } 2^{t-1} < m \text{ so } 2^t < 2m \text{ and } 2^{t+1} < 4m\text{)} \\
&= 8m - 4 + 4m + m \\
&= O(m)
\end{aligned}
$$

Note: For this class, you are not expected to already know facts like $\sum_{k=0}^{t-1} 2^k = 2^t - 1$. If you run across calculations like this in homework, you should feel free to use outside resources (math books, Wolfram Alpha, etc.) to help you solve the problem.