

# CS 88: Security and Privacy

## 10: Introduction to Cryptography

02-27-2024

slides adapted from Dave Levine, Vitaly Shmatikov, Christo Wilson, and Franz Roesner



A CRYPTO NERD'S  
IMAGINATION:

HIS LAPTOP'S ENCRYPTED.  
LET'S BUILD A MILLION-DOLLAR  
CLUSTER TO CRACK IT.

NO GOOD! IT'S  
4096-BIT RSA!

BLAST! OUR  
EVIL PLAN  
IS FOILED!



WHAT WOULD  
ACTUALLY HAPPEN:

HIS LAPTOP'S ENCRYPTED.  
DRUG HIM AND HIT HIM WITH  
THIS \$5 WRENCH UNTIL  
HE TELLS US THE PASSWORD.

GOT IT.



# Cryptography

- Cryptography: An ancient art
  - 500BC – 20<sup>th</sup> century: Design -> break -> repair -> break -> repair ->.....
- Modern Cryptography: Cryptography as a science
  - relies on rigorous threat models
  - firm theoretical foundations and proofs!

# Modern Cryptography

*Design, analysis and implementation of **mathematical techniques** for **securing** information, systems and computation against **adversarial attacks**.*

Modern Cryptography: How many of the following actions involve cryptography ?

1. Git cloning your lab repo
2. Connecting to Swarthmore's WiFi
3. Updating software on your device
4. Making online purchases

- A. One of these
- B. Two of these
- C. Three of these
- D. Four of these

# Modern Cryptography

*If you don't understand what you want to achieve, how can you possibly know when (or if) you have achieved it?*

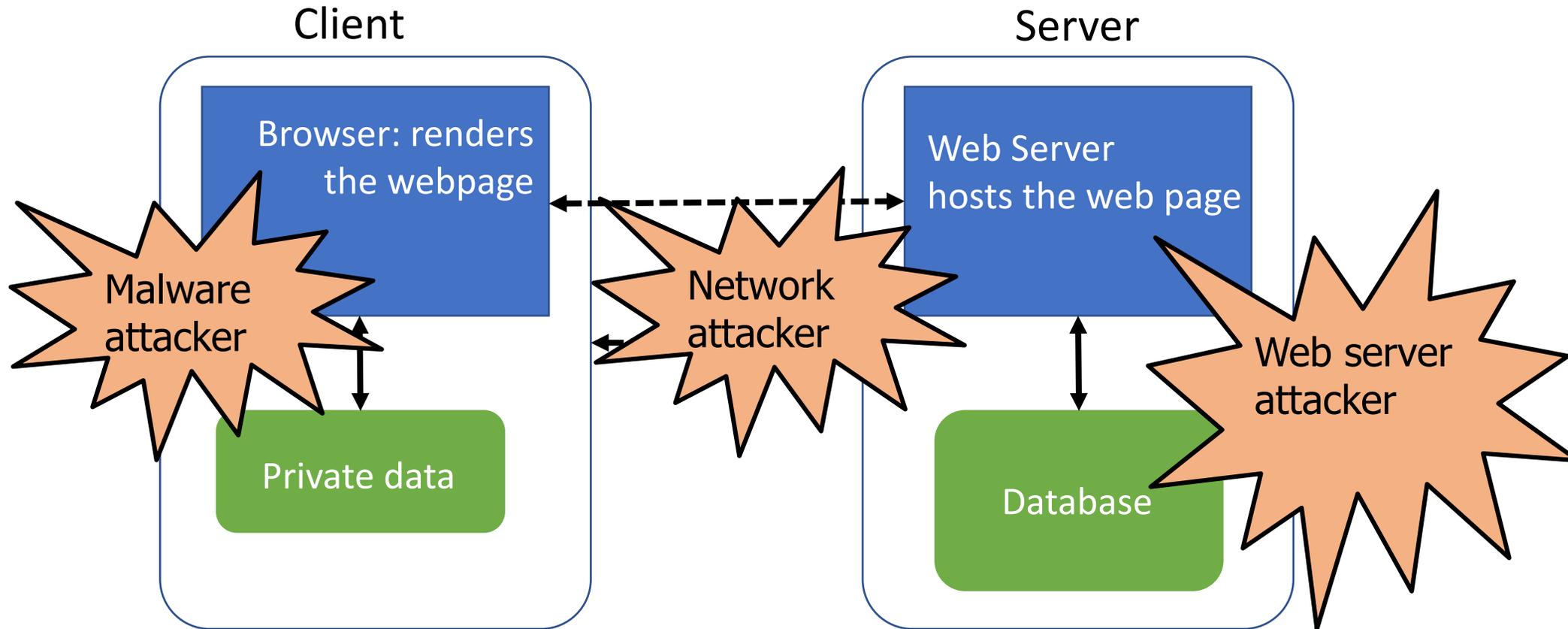
# Modern Cryptography

Importance of clear assumptions:

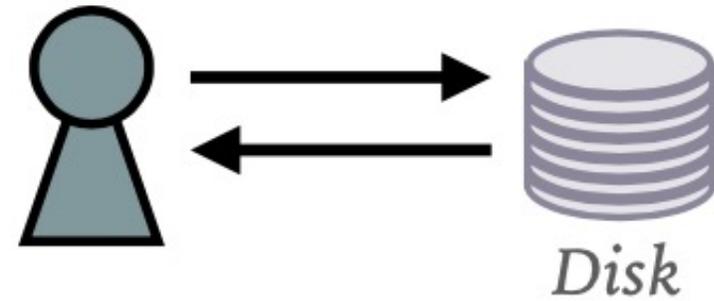
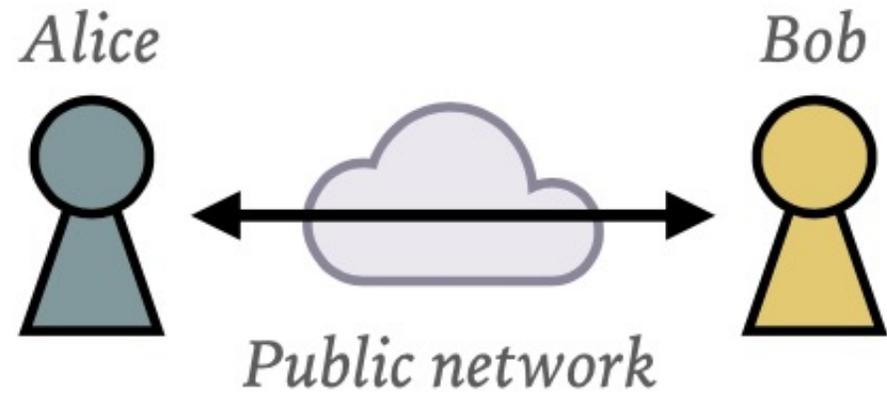
- allows researchers to validate assumptions
- comparison between schemes based on different assumptions
- re-adjust for weaknesses in assumptions

*any new cryptographic construction should be proven secure with respect to a specific definition, and a set of clearly stated assumptions*

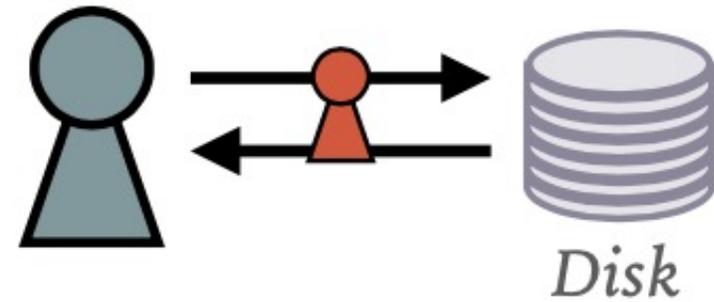
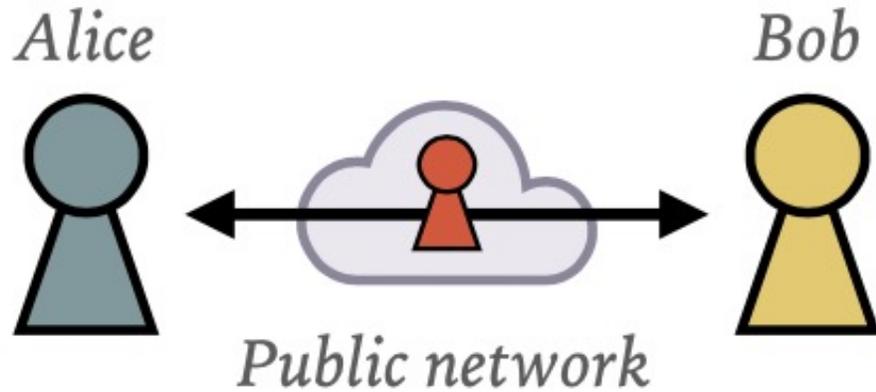
# Where Does the Attacker Live?



# Scenarios and Goals



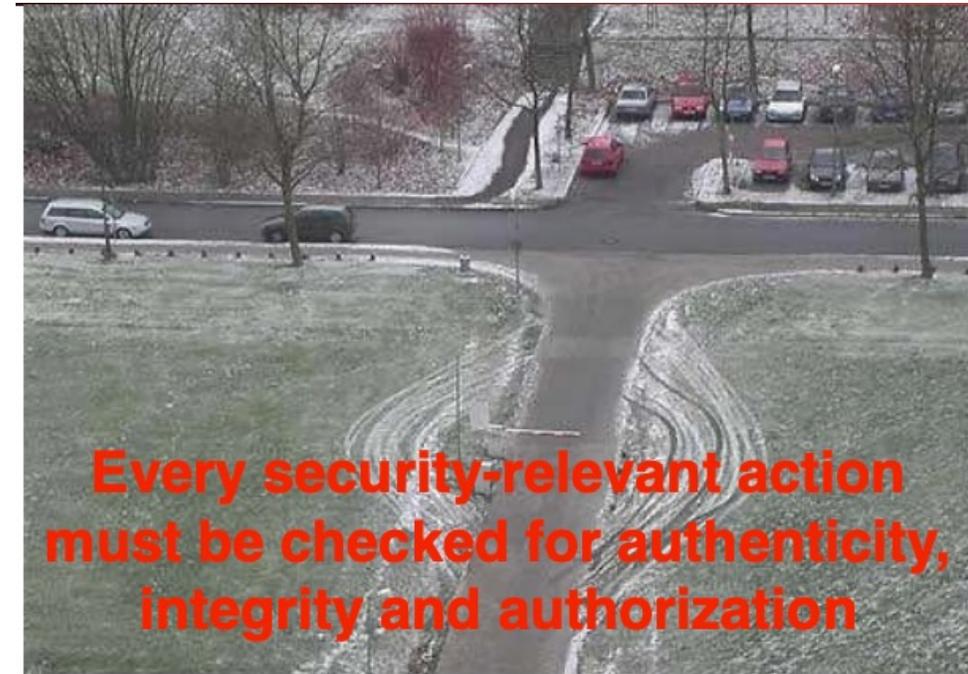
# Scenarios and Goals



- Confidentiality** Keep others from reading Alice's messages/data
- Integrity** Keep others from undetectably tampering with Alice's messages/data
- Authenticity** Keep others from undetectably impersonating Alice (keep her to her word too!)

# Recall the Bigger Picture

- Cryptography: small piece of a larger system
- Protect the entire system (recall: the weakest link)
  - physical security
  - OS security
  - Network security
  - Users
  - Cryptography
- Cryptography is a crucial part of this toolbox



# Cryptography: Terms



- Encryption (E): The process of transforming a message so that its meaning is not obvious
- Decryption (D): The process of transforming an encrypted message back into its original form.
- Plaintext (P): Original, unencrypted form of a message
- Ciphertext (C): The encrypted form of a message

Formal Notation: We seek a cryptosystem for which  $P = D (E (P))$

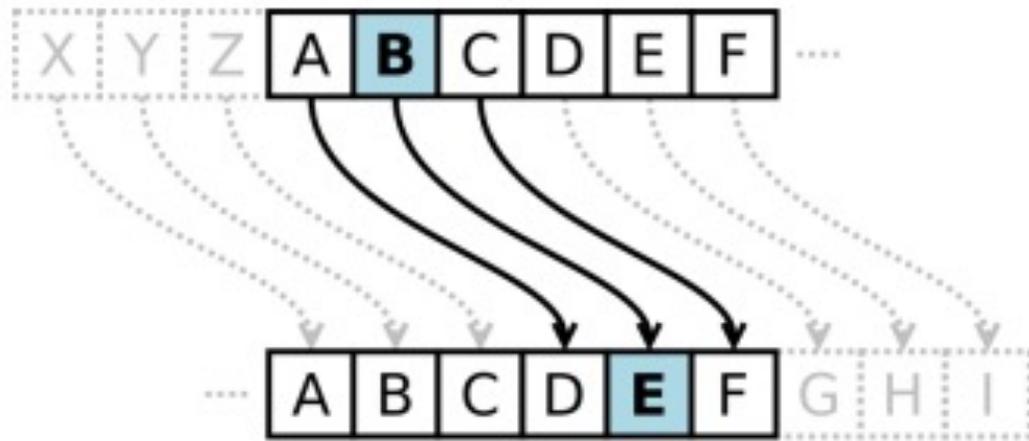
# Historical Ciphers

- Substitution Cipher
  - Monoalphabetic - Ceasar's Cipher – fixed subst. over the entire message
  - Polyalphabetic – a number of substitutions at different positions in the message
- Transposition Ciphers
- Codebooks
- Machines

*Recommended Reading: The Codebreakers by David Kahn, The Code Book by Simon Singh*

# Ceasar Cipher: Substitution Cipher

Plaintext letters replaced with letters fixed shift way in the alphabet.



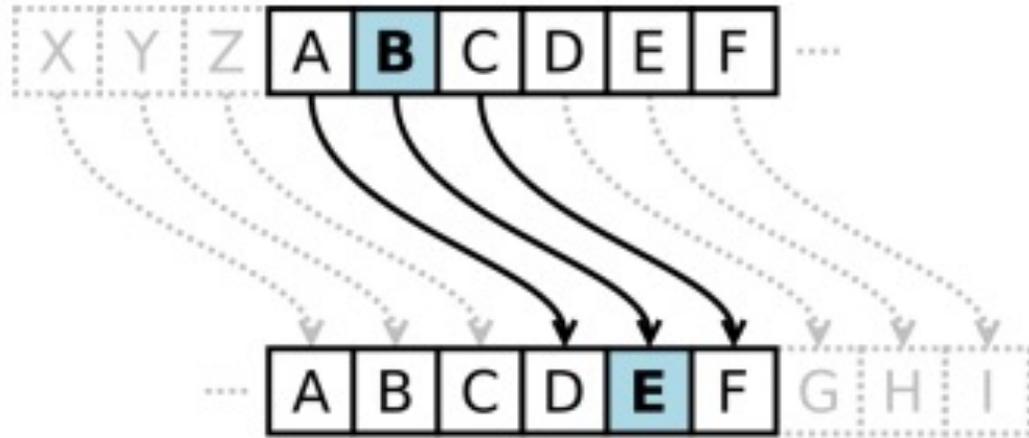
Example:

- Plaintext: HEY BRUTUS BRING A KNIFE TO THE PARTY.
- Ciphertext: **KHB EUXWXV EULQJ D NQLIH WR WKH SDUWB**
- Key Shift 3:
  - ABCDEFGHIJKLMNOPQRSTUVWXYZ
  - DEFPGHIJKLMNOPQRSTUVWXYZABC



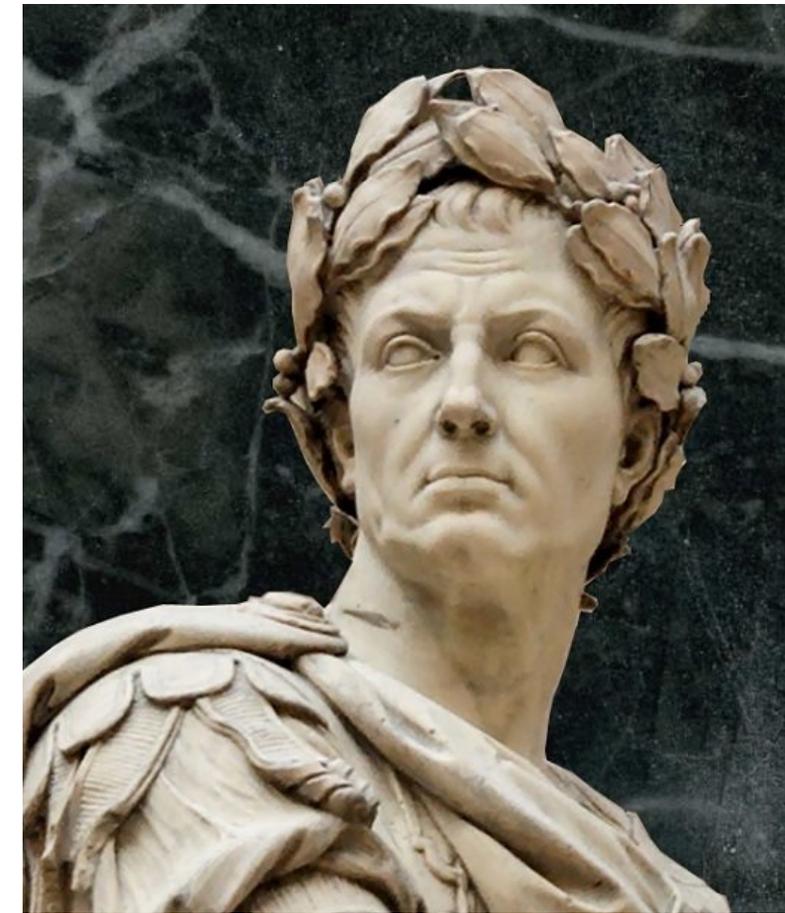
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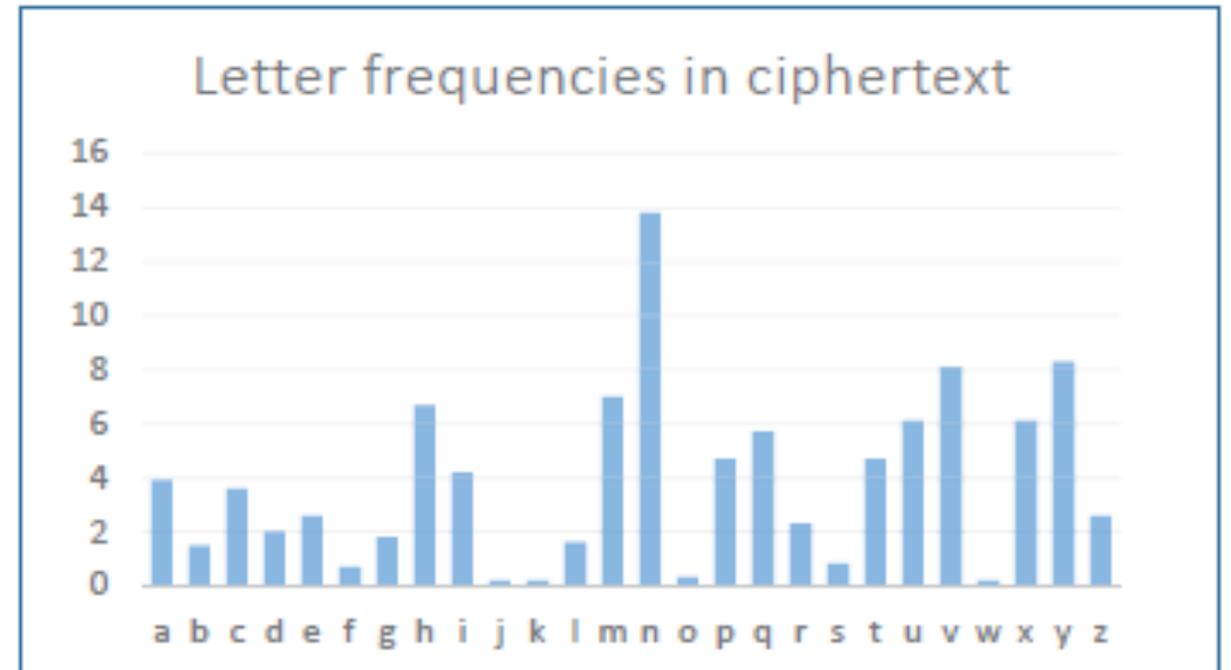
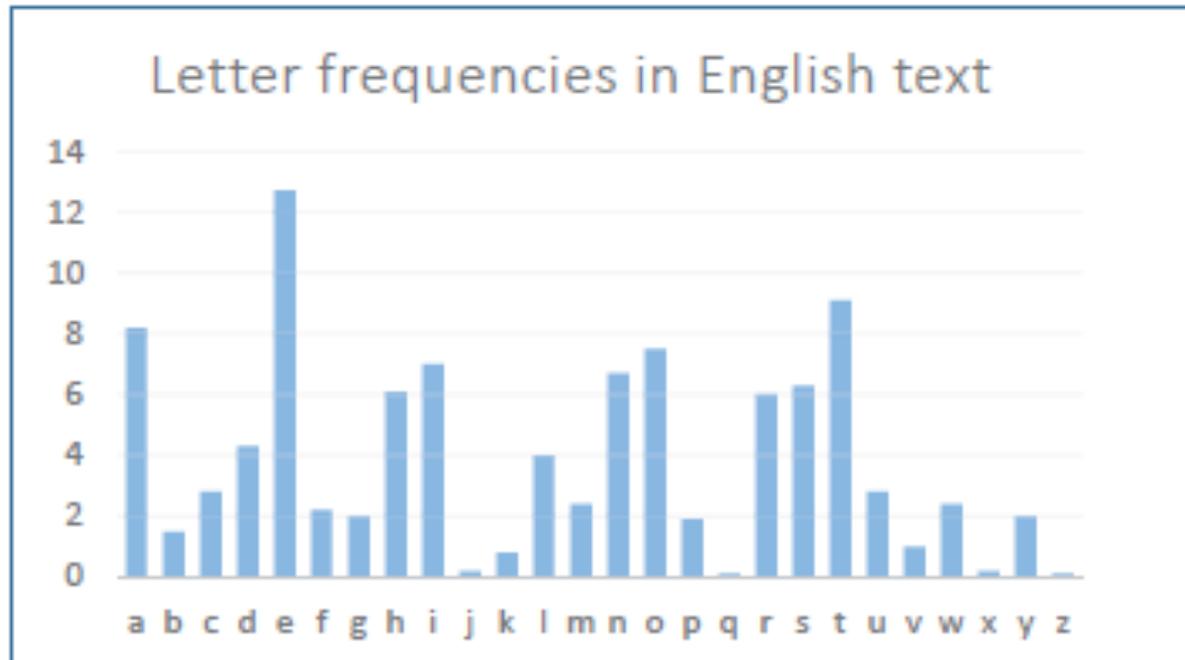
- Encryption and Decryption are symmetric.
- Key space?
  - 26
- Attack shift ciphers?
  - brute force

# Substitution Cipher

- **Superset of shift ciphers:** each letter is substituted for another one.
- One implementation: Add a secret key
- Example
  - Plaintext: ABCDEFGHIJKLMNOPQRSTUVWXYZ
  - Cipher: ZEBRASCDFGHIJKLMNOPQTUVWXY
- “state-of-the-art” for thousands of years

# Monoalphabetic Substitution Cipher

- What is the key space?
  - $26!$  approx. =  $2^{88}$
- Launching an attack?
  - frequency analysis: the study of frequency of letters or groups of letters (grams).
  - Common letters: T, A, E, I, O
  - Common 2-letter combinations (bi-grams): TH, HE, IN, ER
  - Common 3-letter combinations (tri-grams): THE, AND, ING.



# Cryptanalysis of Monoalphabetic Substitution

- Dominates cryptography through the first millennium
- Frequency analysis (Al-Kindi from 800 AD)
- Lessons?
  - Use large blocks: instead of replacing ~6 bits at a time, replace 64 or 128 bits
    - Leads to block ciphers like DES and AES
  - Use different substitutions to prevent frequency analysis
    - Leads to polyalphabetic substitution ciphers and stream ciphers

# Vigenère Cipher (1596)



- Main weakness of monoalphabetic substitution ciphers:
  - Each letter in the ciphertext corresponds to only one letter in the plaintext
- Polyalphabetic substitution cipher
  - Given a key  $K = (k_1, k_2, \dots, k_m)$
  - Shift each letter  $p$  in the plaintext by  $k_i$ , where  $i$  is modulo  $m$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Plaintext    CRYPTOGRAPHY  
Key         LUCKLUCKLUCK (Shift 11 20 2 10 11 20 2 11 ...)  
Ciphertext   NLAZEIIBLJJI

# Kasisky Test

Plaintext	T H E S U N A N D	T H E	M A N I N	T H E	M O O N
Key	K I N G K I N G K	I N G	K I N G K	I N G	K I N G
Ciphertext	D P R Y E V N T N	B U K	W I A O X	B U K	W W B T

  
Distance = 8

- Repeating patterns (of length  $>2$ ) in ciphertext are a tell
  - Likely due to repeated plaintext encrypted under repeated key characters
  - The distance is likely to be a multiple of the key length

# Cryptanalysis of Vigenère Cipher

- Cracking Vigenère (1854 or 1863)
  1. Guess the key length  $x$  using Kasisky test or index of coincidence
  2. Divide the ciphertext into  $x$  shift cipher encryptions
  3. Use frequency analysis on each shift cipher
- Lessons?
  - As key length increases, letter frequency becomes more random
  - If key never repeated, Vigenère wouldn't be breakable!

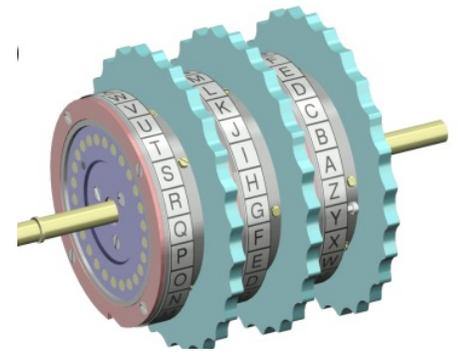




- WW2 German Enigma machine
  - Polyalphabetic substitution cipher
    - Substitution table changes from character to character
    - Rotors control substitutions
- Allies broke Enigma (even before the war), significant intelligence impact
- Computers were built to break WW2 ciphers, by Alan Turing and others



# Enigma Machine



- Use rotors that change position after each key
- Key: initial setting of the rotors
- Key space?
  - $26^n$  for  $n$  rotors
- KeyGen:
  - Choose rotors, rotor orders, rotor positions, and plugboard settings
  - 158,962,555,217,826,360,000 possible keys!

# Cryptanalysis: Enigma

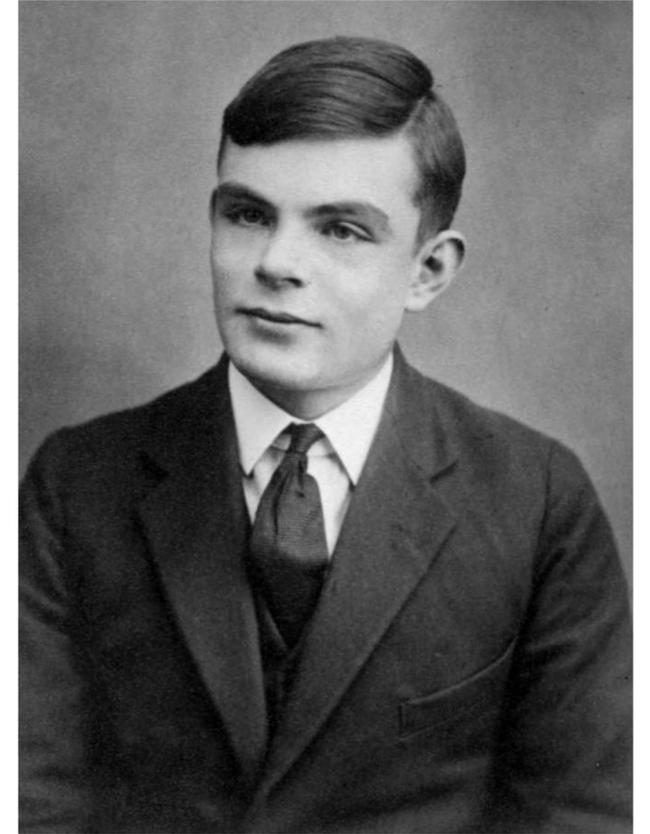
- Polish and British cryptographers built BOMBE, a machine to brute-force Enigma keys
- Why was Enigma breakable?
  - **Kerckhoff's principle**: The Allies stole Enigma machines, so they knew the algorithm
  - **Known plaintext attacks**: the Germans often sent predictable messages (e.g. the weather report every morning)
  - **Chosen plaintext attacks**: the Allies could trick the Germans into sending a message (e.g. "soldiers at Normandy")
  - **Brute-force**: BOMBE would try many keys until the correct one was found



BOMBE machine

# Legacy of Enigma

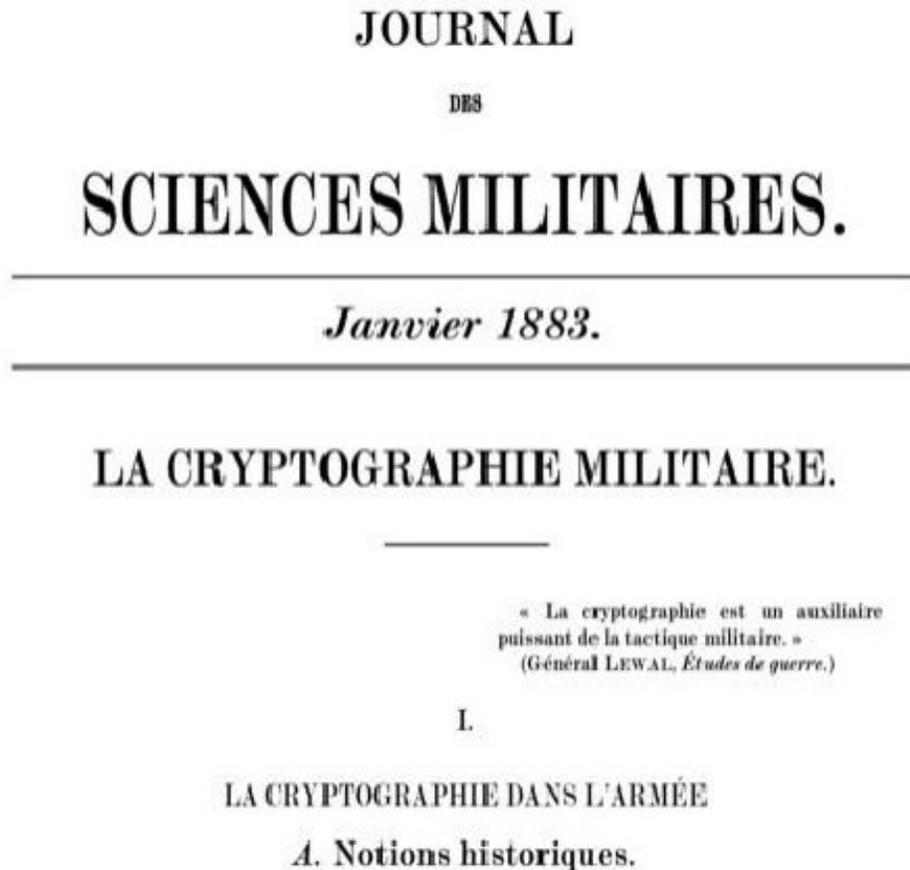
- Alan Turing, one of the cryptographers who broke Enigma, would go on to become one of the founding fathers of computer science
- Most experts agree that the Allies breaking Enigma shortened the war in Europe by about a year



Alan Turing

# Kerckhoffs' principle

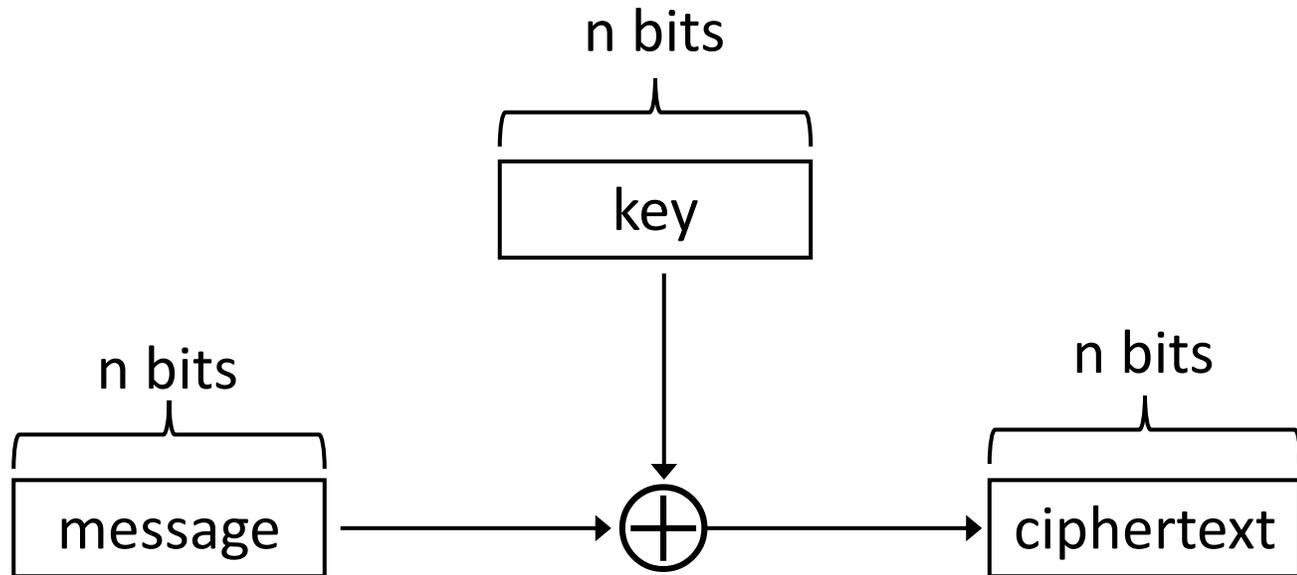
Encryption and Decryption and Key Generation Algorithm are publicly known. *The only unknown is the shared secret key*



Wikipedia

# One Time Pad (1920s)

- Fix vulnerability in Vigenère cipher: use very long keys
- Key is a random string: **at least as long as the plaintext**
- Plaintext: Message that is n bits long
- Key:  $\{0, 1\}^n$  sequence of n bits chosen uniformly at random.



# Review: XOR

The XOR operator takes two bits and outputs one bit

$0 \oplus 0 = 0$
$0 \oplus 1 = 1$
$1 \oplus 0 = 1$
$1 \oplus 1 = 0$

Useful properties of XOR

$$x \oplus 0 = x$$

$$x \oplus x = 0$$

$$x \oplus y = y \oplus x$$

$$(x \oplus y) \oplus z = x \oplus (y \oplus z)$$

$$(x \oplus y) \oplus x = y$$

# Review: XOR Algebra

Algebra works on XOR too

$y \oplus 1 = 0$	Goal: Solve for y
$y \oplus 1 \oplus 1 = 0 \oplus 1$	XOR both sides by 1
$y = 1$	Simplify with identities

# One-Time Pads: Key Generation

Alice

$K$	0	1	1	0	0	1	0	1	0	1	1	1
-----	---	---	---	---	---	---	---	---	---	---	---	---

The key  $K$  is a randomly-chosen bitstring.

Recall: We are in the symmetric-key setting, so we'll assume Alice and Bob both know this key.

# One-Time Pads: Encryption

Alice

$K$	0	1	1	0	0	1	0	1	0	1	1	1
$M$	1	0	0	1	1	0	0	1	0	1	0	0

The plaintext  $M$  is the bitstring that Alice wants to encrypt.

Idea: Use XOR to scramble up  $M$  with the bits of  $K$ .

# One-Time Pads: Encryption

Alice												
$K$	0	1	1	0	0	1	0	1	0	1	1	1
	$\oplus$											
$M$	1	0	0	1	1	0	0	1	0	1	0	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
$C$	1	1	1	1	1	1	0	0	0	0	1	1

Encryption algorithm: XOR each bit of  $K$  with the matching bit in  $M$ .

The ciphertext  $C$  is the encrypted bitstring that Alice sends to Bob over the insecure channel.

# One-Time Pads: Decryption

Bob

$K$	0	1	1	0	0	1	0	1	0	1	1	1
$C$	1	1	1	1	1	1	0	0	0	0	1	1

Bob receives the ciphertext  $C$ . Bob knows the key  $K$ . How does Bob recover  $M$ ?

# One-Time Pads: Decryption

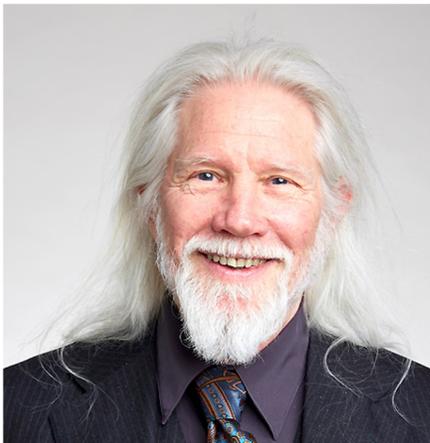
Bob

<i>K</i>	0	1	1	0	0	1	0	1	0	1	1	1
	$\oplus$											
<i>C</i>	1	1	1	1	1	1	0	0	0	0	1	1
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
<i>M</i>	1	0	0	1	1	0	0	1	0	1	0	0

Decryption algorithm: XOR each bit of *K* with the matching bit in *C*.

# Cryptography by Computers

- The modern era of cryptography started after WWII, with the work of Claude Shannon
- “New Directions in Cryptography” (1976) showed how number theory can be used in cryptography
  - Its authors, Whitfield Diffie and Martin Hellman, won the Turing Award in 2015 for this paper
- *This is the era of cryptography we’ll be focusing on.*



# How Cryptosystems work today

- Layered approach:
  - Cryptographic protocols (e.g., CBC mode encryption)
  - Built on: Cryptographic primitives (block ciphers)
- Flavors of cryptography:
  - Symmetric: private key
  - Asymmetric: public key
- Public algorithms: Kerckhoff's principle
- *Security proofs based on assumptions (not this course)*
- **Warning! Here be dragons!**
  - careful about inventing your own
  - Use vetted libraries to apply crypto algorithms



# Cryptosystem Stack

- Primitives:
  - AES/DES
  - ESA / ElGamal / Elliptic Curve
- Modes
  - Block mode (CBC, ECV, CTR, GCM..)
  - Padding structures
- Protocols:
  - TLS, SSL, SSH
- Usage of protocols:
  - Browser security
  - Secure remote logins

# Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string  $K$ , called the key.
- Asymmetric cryptography
  - Each party creates a public key  $p_k$  and a secret key  $s_k$
  - Inventors won Turing Award!

# Kerckhoff's Principle

- Security of a cryptographic object should depend only on the secrecy of the secret (private) key.
- Security should not depend on the secrecy of the algorithm itself.
- Foreshadow: Need for randomness – the key to keep private