CS 88: Security and Privacy

12: Symmetric Key Cryptography

10-06-2022

slides adapted from Dave Levine, Jonathan Katz and David Wagner

SWARTHMORE COLLEGE

Modern Cryptography

Design, analysis and implementation of mathematical techniques for securing information, systems and computation against adversarial attacks.

Modern Cryptography

If you don't understand what you want to achieve, how can you possibly know when (or if) you have achieved it?

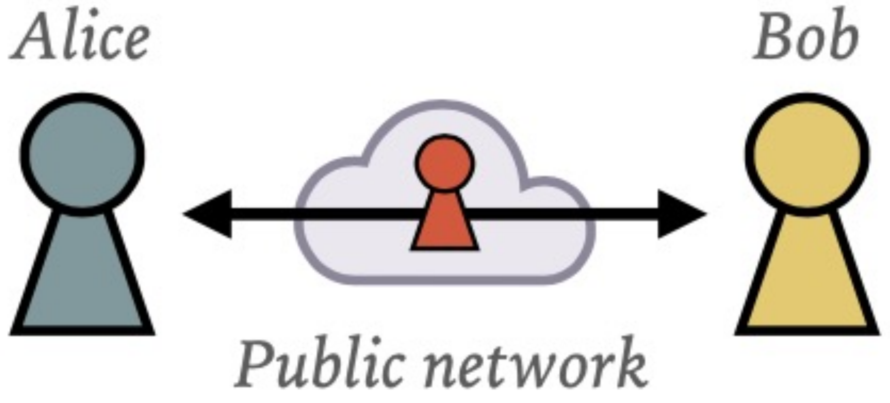
Modern Cryptography

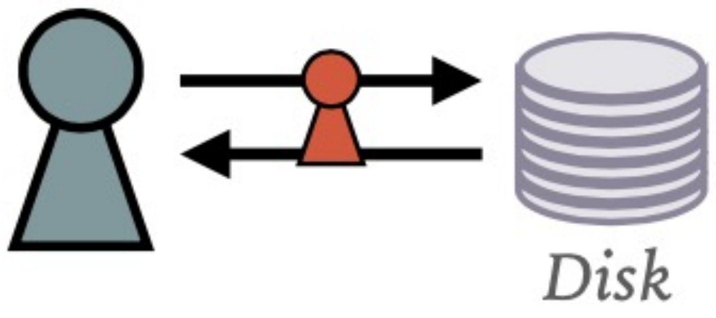
Importance of clear assumptions:

- allows researchers to validate assumptions
- comparison between schemes based on different assumptions
- re-adjust for weaknesses in assumptions

any new cryptographic construction should be proven secure with respect to a specific definition, and a set of clearly stated assumptions

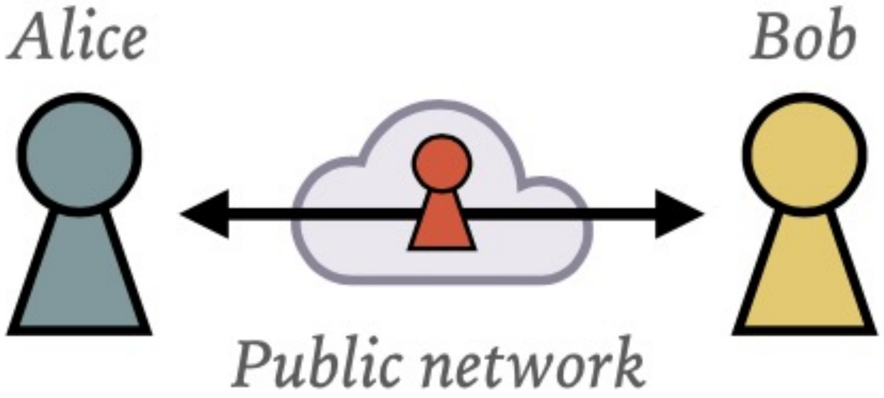
Scenarios and Goals

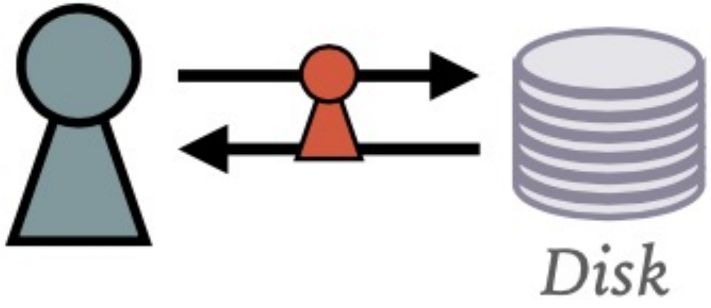




Confidentiality	Keep others from reading Alice's messages/data
Integrity	Keep others from undetectably tampering with Alice's messages/data
Authenticity	Keep others from undetectably impersonating Alice (keep her to her word too!)

Cryptography: Terms



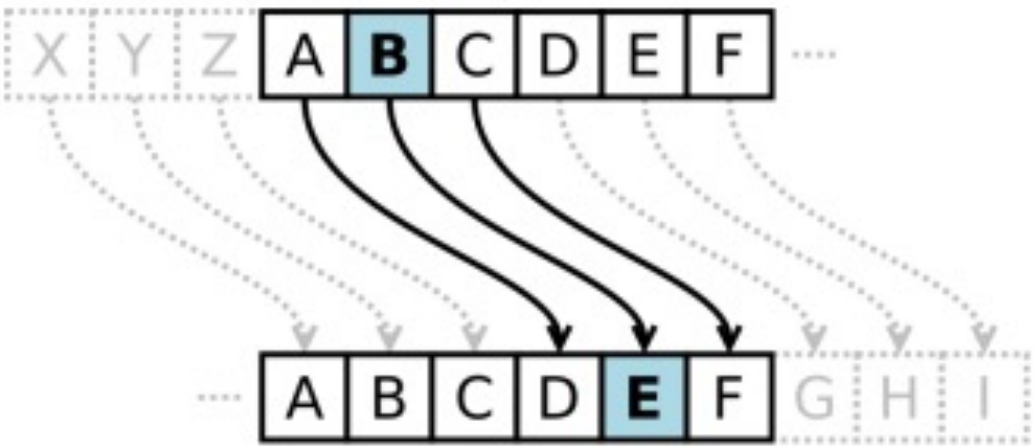


Encryption (E): The process of transforming a message so that its meaning is not obvious
Decryption (D): The process of transforming an encrypted message back into its original form.
Plaintext (m): Original, unencrypted form of a message
Ciphertext (p): The encrypted form of a message

Formal Notation: We seek a cryptosystem for which P = D (E (P))

Ceasar Cipher: Substitution Cipher

Plaintext letters replaced with letters fixed shift way in the alphabet.



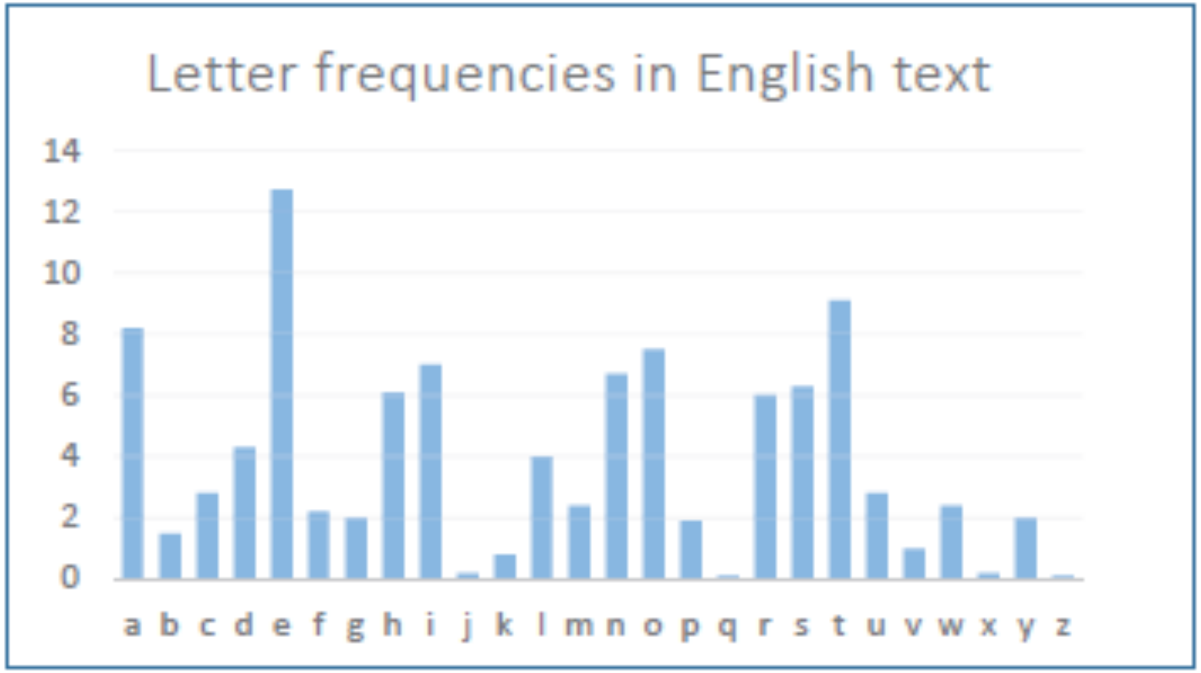
Example:

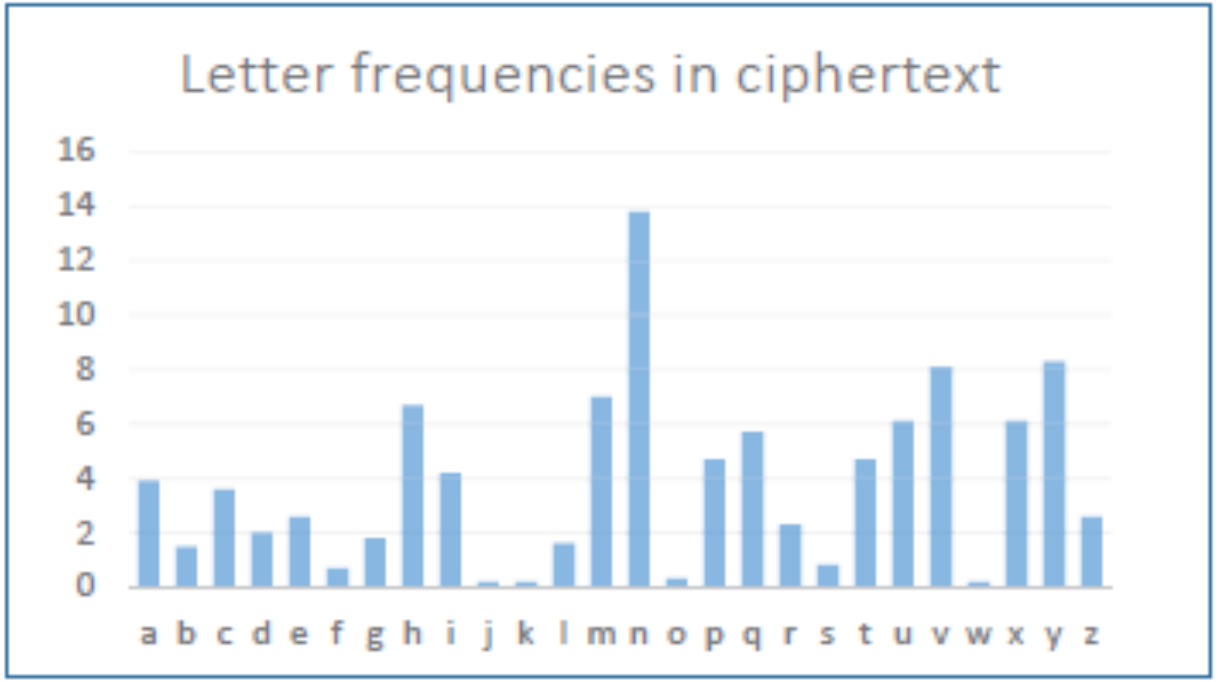
- Plaintext: HEY BRUTUS BRING A KNIFE TO THE PARTY.
- Ciphertext: KHB EUXWXV EULQJ D NQLIH WR WKH SDUWB
- Key Shift 3:
  - ABCDEFGHIJKLMNOPQRSTUVWXYZ
  - DEFGHIJKLMNOPQRSTUVWXYZABC

- Encryption and Decryption are symmetric.
- Key space?
  - 26
- Attack shift ciphers?
  - brute force

Monoalphabetic Substitution Cipher

- What is the key space?
  - 26
- Launching an attack?
  - frequency analysis: the study of frequency of letters or groups of letters (grams).
  - Common letters: T, A, E, I, O
  - Common 2-letter combinations (bi-grams): TH, HE, IN, ER
  - Common 3-letter combinations (tri-grams): THE, AND, ING.





Kerckhoffs' principle

Encryption and Decryption and Key Generation Algorithm are publicly known. *The only unknown is the shared secret key*

JOURNAL

DES

SCIENCES MILITAIRES.

Janvier 1883.

LA CRYPTOGRAPHIE MILITAIRE.

« La cryptographie est un auxiliaire puissant de la tactique militaire. »
(Général LEWAL, *Études de guerre.*)

I.

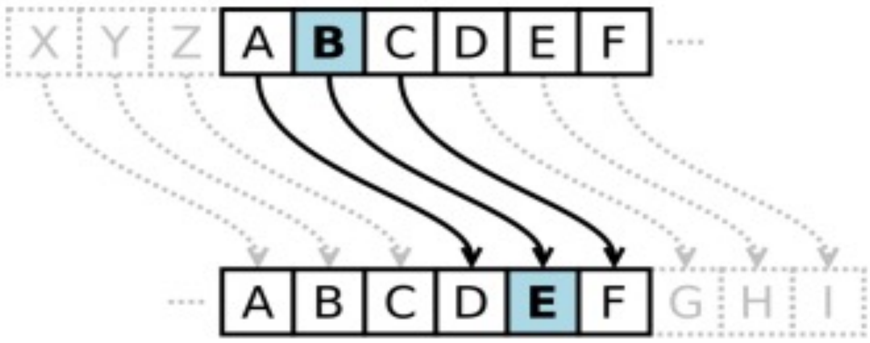
LA CRYPTOGRAPHIE DANS L'ARMÉE

A. Notions historiques.

Wikipedia

Ceasar Cipher: Substitution Cipher

Plaintext letters replaced with letters fixed shift way in the alphabet.



Lessons Learnt?

- Simple, exhaustive key search can be effective
- Key space needs to be large enough to prevent attack
- Use different substitutions to prevent frequency analysis

Vigenère Cipher (1596)

- Main weakness of monoalphabetic substitution ciphers:
  - Each letter in the ciphertext corresponds to only one letter in the plaintext
- Polyalphabetic substitution cipher
  - Given a key $K = (k_1, k_2, \ldots, k_m)$
  - Shift each letter p in the plaintext by k_i, where i is modulo m

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Plaintext	CRYPTOGRAPHY
Key	LUCKLUCKLUCK (Shift 11 20 2 10 11 20 2 11 …)
Ciphertext	NLAZEIIBLJJI

Kasisky Test Index of coincidence

Plaintext	T	H	E	S	U	N	A	N	D	T	H	E	M	A	N	I	N	T	H	E	M	O	O	N
Key	K	I	N	G	K	I	N	G	K	I	N	G	K	I	N	G	K	I	N	G	K	I	N	G
Ciphertext	D	P	R	Y	E	V	N	T	N	B	U	K	W	I	A	O	X	B	U	K	W	W	B	T

Distance = 8

- Repeating patterns (of length >2) in ciphertext are a tell
  - Likely due to repeated plaintext encrypted under repeated key characters
  - The distance is likely to be a multiple of the key length

Vigenère Cipher (1596)

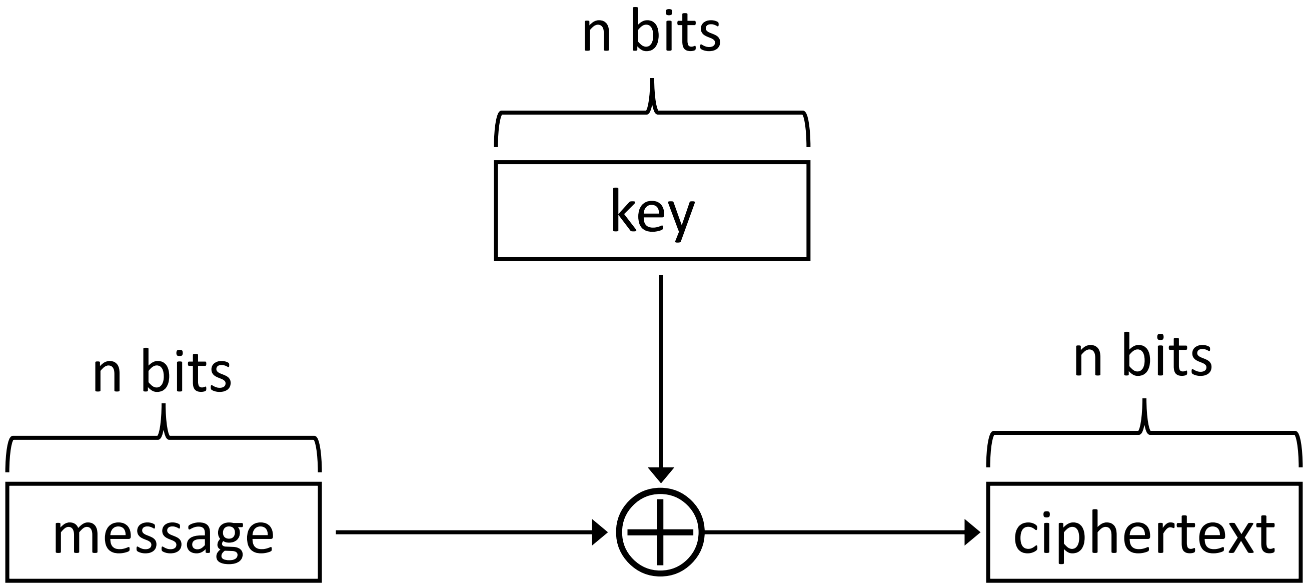
- Lessons learnt?
  - As key length increases, letter frequency becomes more random
  - If key never repeated, Vigenère wouldn't be breakable!

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Plaintext	CRYPTOGRAPHY	
Key	LUCKLUCKLUCK	(Shift 11 20 2 10 11 20 2 11 …)
Ciphertext	NLAZEIIBLJJI	

One Time Pad (1920s)

- Fix vulnerability in Vigenère cipher: use very long keys
- Key is a random string: at least as long as the plaintext
- Plaintext: Message that is n bits long
- Key: $\{0, 1\}^n$ sequence of n bits chosen uniformly at random.



Review: XOR

The XOR operator takes two bits and outputs one bit

$0 \oplus 0 = 0$
$0 \oplus 1 = 1$
$1 \oplus 0 = 1$
$1 \oplus 1 = 0$

Useful properties of XOR

$x \oplus 0 = x$
$x \oplus x = 0$
$x \oplus y = y \oplus x$
$(x \oplus y) \oplus z = x \oplus (y \oplus z)$
$(x \oplus y) \oplus x = y$

Review: XOR Algebra

Algebra works on XOR too

$y \oplus 1 = 0$	Goal: Solve for y
$y \oplus 1 \oplus 1 = 0 \oplus 1$	XOR both sides by 1
$y = 1$	Simplify with identities

One-Time Pads: Key Generation

Alice

K	0	1	1	0	0	1	0	1	0	1	1	1

The key *K* is a randomly-chosen bitstring.

Recall: We are in the symmetric-key setting, so we'll assume Alice and Bob both know this key.

One-Time Pads: Encryption

Alice

K	0	1	1	0	0	1	0	1	0	1	1	1
M	1	0	0	1	1	0	0	1	0	1	0	0

The plaintext M is the bitstring that Alice wants to encrypt.

Idea: Use XOR to scramble up M with the bits of K.

One-Time Pads: Encryption

Alice

K	0	1	1	0	0	1	0	1	0	1	1	1
	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
M	1	0	0	1	1	0	0	1	0	1	0	0
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
C	1	1	1	1	1	1	0	0	0	0	1	1

Encryption algorithm: XOR each bit of K with the matching bit in M.

The ciphertext C is the encrypted bitstring that Alice sends to Bob over the insecure channel.

One-Time Pads: Decryption

Bob

K	0	1	1	0	0	1	0	1	0	1	1	1
C	1	1	1	1	1	1	0	0	0	0	1	1

Bob receives the ciphertext C. Bob knows the key K. How does Bob recover M?

One-Time Pads: Decryption

Bob

K	0	1	1	0	0	1	0	1	0	1	1	1
	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
C	1	1	1	1	1	1	0	0	0	0	1	1
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
M	1	0	0	1	1	0	0	1	0	1	0	0

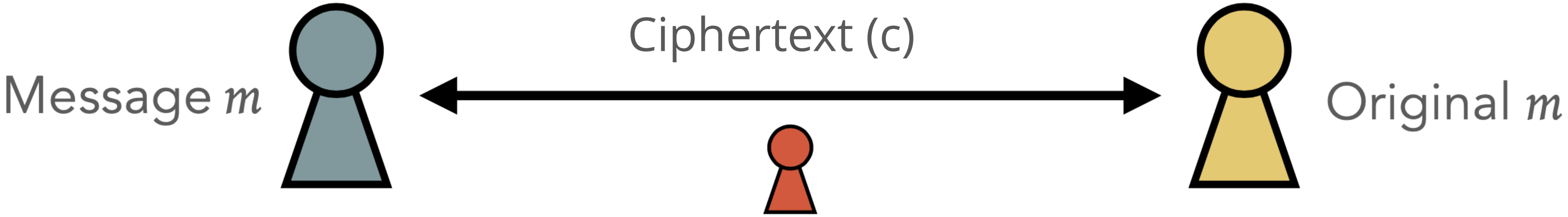
Decryption algorithm: XOR each bit of K with the matching bit in C.

Modern Cryptography

any new cryptographic construction should be proven secure with respect to a specific definition, and a set of clearly stated assumptions

Threat Models

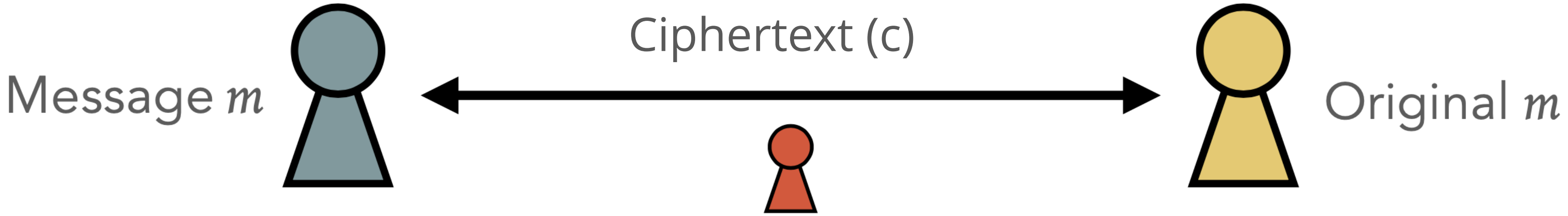
- Ciphertext-only attack: An attacker (Eve) observes ciphertext and nothing else
  - Can Eve observe more than one ciphertext?
  - this distinction can make a big difference!



Ciphertext: c

Threat Models

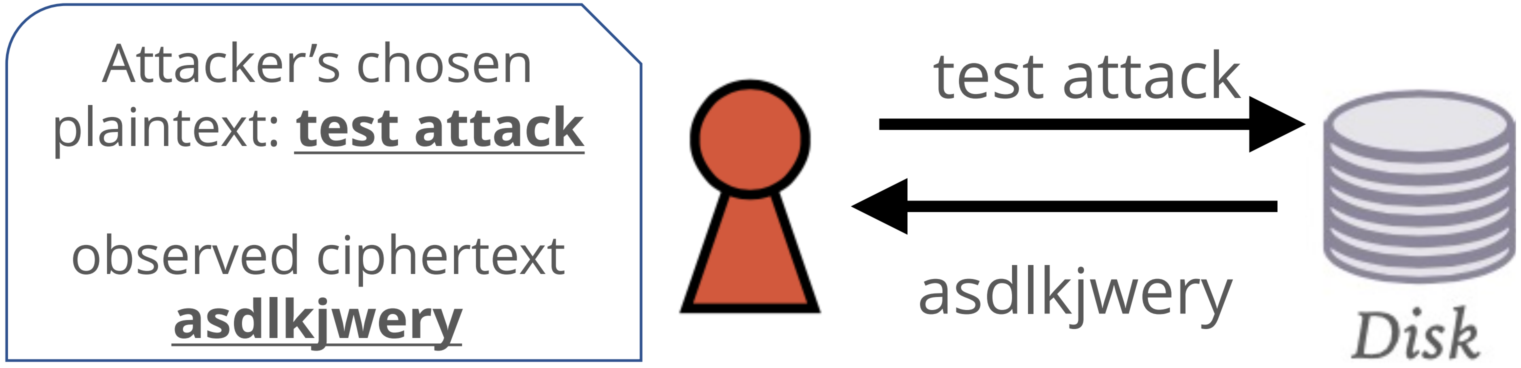
- Known-Plaintext Attack: An attacker (Eve) observes ciphertext and knows underlying plaintext
  - e.g., Alice: plaintext: Hello! ciphertext: 23asdf1941
    - Bob: plaintext: Hello! ciphertext: 23asdf1941



Alice's plaintext: **Hello!**
ciphertext: **23asdf1941**

Threat Models

- Chosen-Plaintext Attack:
  - Observe one or more ciphertext, where plaintext is unknown
  - Also observe ciphertext for plaintext of attacker's choosing.

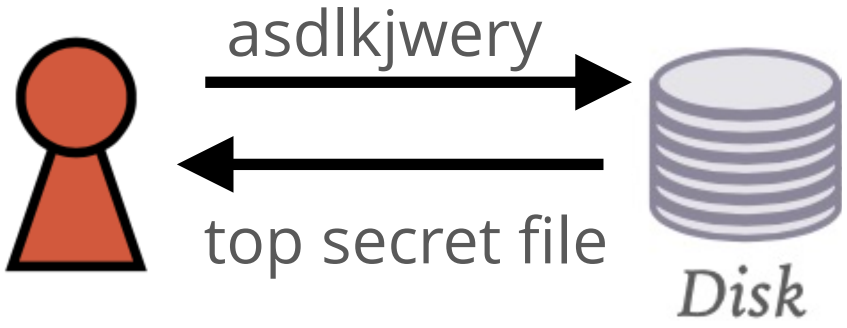


Threat Models

- Chosen-Ciphertext Attack:
  - Attacker is able to get the parties to decrypt certain cipher texts of that attacker's choice.

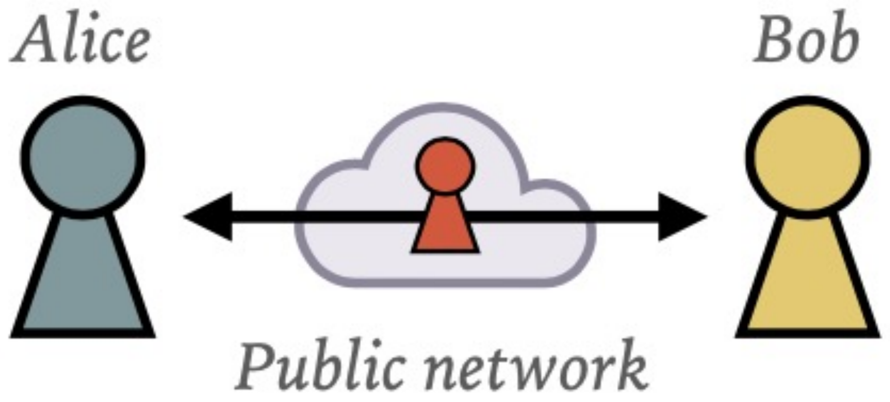
Attacker’s chosen ciphertext:
asdlkjwery

observed plaintext:
top secret attack



A Perfectly Secure Encryption Scheme

Regardless of any prior information the attacker has about the plaintext
the ciphertext observed by the attacker
should leak no additional information about the plaintext.



<u>Alice can only observe one ciphertext going over the network</u>

A Secure Encryption Scheme

An encryption scheme given by: (key gen alg., encryption alg, decryption alg.) over message space ***M*** is perfectly secure iff

$\forall$ probability distribution over **M**

$\forall$ message $m \in \mathbf{M}$

$\forall$ ciphertext $c \in \mathbf{C}$ for which $\Pr[\mathbf{C} = c] > 0$

we have

$$\Pr[\mathbf{M} = m \mid \mathbf{C} = c] = \Pr[\mathbf{M} = m]$$

One Time Pad: Perfectly Secure?

- OTP achieves Perfect Secrecy
  - Shannon or Information Theoretic Security
  - Basic idea: ciphertext reveals no "additional information" about plaintext

Proving Perfect Security: One Time Pads

Problem Statement:

- Suppose Alice has sent one of two messages M_0 or M_1, and Eve has no idea which was sent.
- Eve tries to guess which was sent by looking at the ciphertext.

To Show:

- Eve's probability of guessing correctly is ½
- This is no different than it would be if she had not intercepted the ciphertext at all.

Proving Perfect Security: One Time Pads

Alice randomly chooses a bit string $\in \{0, 1\}^{n}$, and Alice sends the encryption of M_b.
If Eve observes that the ciphertext has some specific value C, what is the conditional probability that b = the input bit string given her observation?

Fix arbitrary distribution over $\mathbf{M} = \{0,1\}^n$, and arbitrary $m, c \in \{0,1\}^n$

$\Pr[M = m \mid C = c] = ?$

$= \Pr[C = c \mid M = m] \cdot \Pr[M = m]/\Pr[C = c]$

$\Pr[M = m \mid C = c] = ?$

$= \Pr[C = c \mid M = m] \cdot \Pr[M = m]/\Pr[C = c]$

$= \Pr[K = m \oplus c] \cdot \Pr[M = m] / 2^{-n}$

$= 2^{-n} \cdot \Pr[M = m] / 2^{-n}$

$= \Pr[M = m]$

$\Pr[C = c]$

$= \Sigma_{m'} \Pr[C = c \mid M = m'] \cdot \Pr[M = m']$

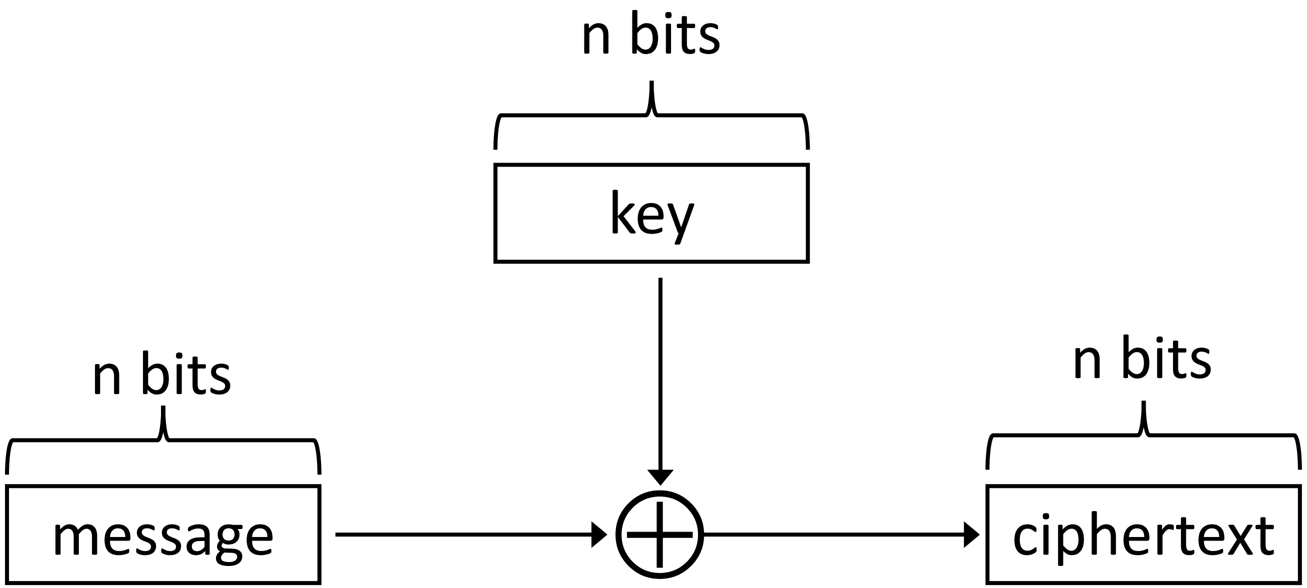
$= \Sigma_{m'} \Pr[K = m' \oplus c] \cdot \Pr[M = m']$

$= \Sigma_{m'} 2^{-n} \cdot \Pr[M = m']$

$= 2^{-n}$

One Time Pad: Limitations

- *The key is as long as the message*
- Only secure if each key is used to encrypt a single message
- Parties must share keys of (total) length = the (total) length of all the messages they might ever send!



Using the same key twice?

Say $c_1 = k \oplus m_1$

$c_2 = k \oplus m_2$

Attacker can compute

$$c_1 \oplus c_2 = (k \oplus m_1) \oplus (k \oplus m_2) = m_1 \oplus m_2$$

This leaks information about m_1, m_2!

Limitations of Perfect Security

Regardless of any prior information the attacker has about the plaintext the ciphertext observed by the attacker should leak no additional information about the plaintext.

- The key is as long as the message
- Only secure if each key is used to encrypt a single message

Limitations are not only of One Time Pads, but inherent to any perfectly secure encryption scheme.

Assumes the attacker as unlimited computational power

Computational Security/Secrecy

Would be okay if a scheme leaked information with a tiny probability to eavesdroppers with bounded computational resources.

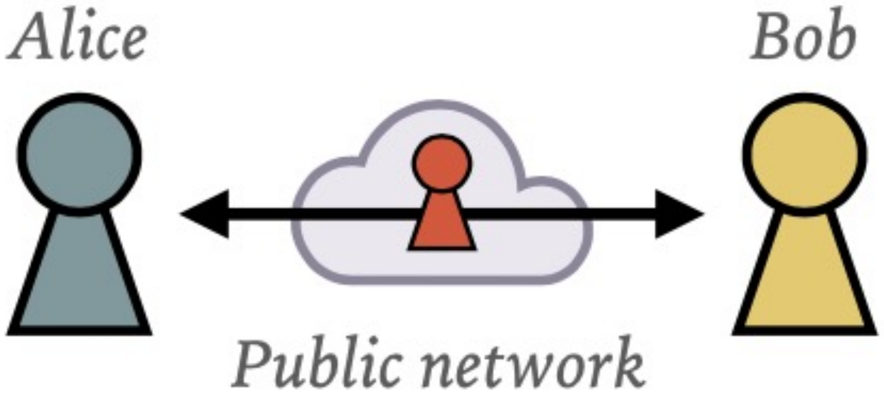
- Allowing security to fail with a tiny probability (negligible in key length n)
  - how tiny is tiny? 2^{-60} : probability of an event occurring every 100 billion years!
- Only consider efficient attackers (bounded in polynomial time by key length)
  - attackers that can brute-force the key space in bounded time.
  - try testing 2^{112} keys? Would take a supercomputer since Big Bang!
  - modern key space? 2^{128} or more!

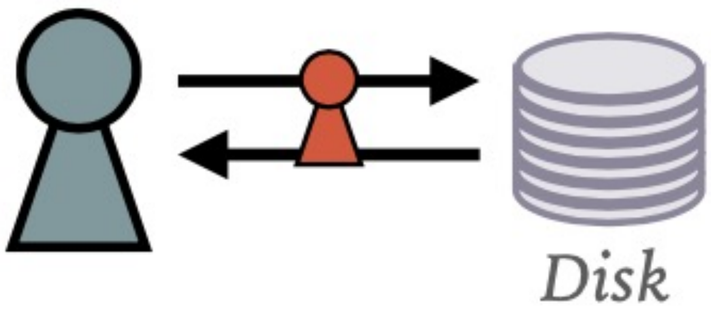
Computational Secrecy: One Time Pads

Key Insight: Randomness –

- something an adversary won’t know, can’t predict and can’t figure out.

Randomness





Explicit Uses of Randomness:

- Generate secret cryptographic keys
- Generate random initialization vectors or nonces for encryption

Use cases

- Generate passwords for new users
- Shuffle the order of votes in an electronic voting machine
- Shuffle cards etc. (for online games)

What does “random” mean?

- What does “uniform” mean?
- Which of the following is a uniform string?
  - 0101010101010101
  - 0010111011100110
  - 0000000000000000
- If we generate a uniform string, each of the above occurs with probability 2^{-16}

What does "random" mean?

- "Randomness" is not a property of a string, but a property of a distribution.

- The uniform distribution on n-bit strings is the distribution U_n where
  - $U_n(x) = 2^{-n}$ for all $x \in \{0,1\}^n$

What does “pseudorandom” mean?

- Informal: cannot be distinguished from uniform (i.e., random)
- Which of the following is pseudorandom?
  - 0101010101010101
  - 0010111011100110
  - 0000000000000000
- Pseudorandomness is a property of a distribution

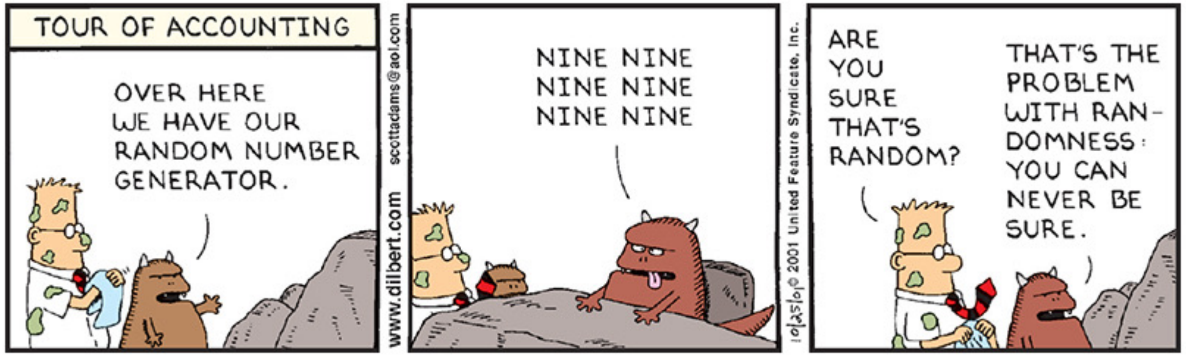
How Random is "Random"?

OXFFFFFFFF EVERY TIME IS OXDEADBEEF —

How a months-old AMD microcode bug destroyed my weekend [UPDATED]

AMD shipped Ryzen 3000 with a serious microcode bug in its random number generator.

JIM SALTER - 10/29/2019, 7:00 AM



› Uptime | IT and business computing insights

Crypto shocker: four of every 1,000 public keys provide no security (updated)

By Dan Goodin | Published 7 days ago

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Flaw Found in an Online Encryption Method

JOHN MARKOFF

ublished: February 14, 2012

SAN FRANCISCO — A team of European and American mathematicians and cryptographers have discovered an unexpected weakness in the encryption system widely used worldwide for online shopping, banking, e-mail and other Internet services intended to remain private and secure.

RECOMMEND
TWITTER
LINKEDIN
COMMENTS (127)
E-MAIL

C’s built-in rand() function

```
unsigned long int next = 1;
/* rand: return pseudo-random integer on 0…32767 */
int rand(void){
   next = next * 11-3515245 + 12345;
   return (unsigned int) (next/65536) % 32768;
}
/* srand: set seed for rand() */
void srand(unsigned int seed){
    next = seed;
}
```

"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin.” -- John von Neumann

Random-number generation

- Two steps:
  1. Continually collect a “pool” of high-entropy (i.e., “unpredictable”) data from external inputs
     1. Delays between network events
     2. Hard-disk access times
     3. Keystroke/mouse movements
  2. When random bits are requested, process this data to generate a sequence of uniform, independent bits/bytes
     - May “block” if insufficient entropy available

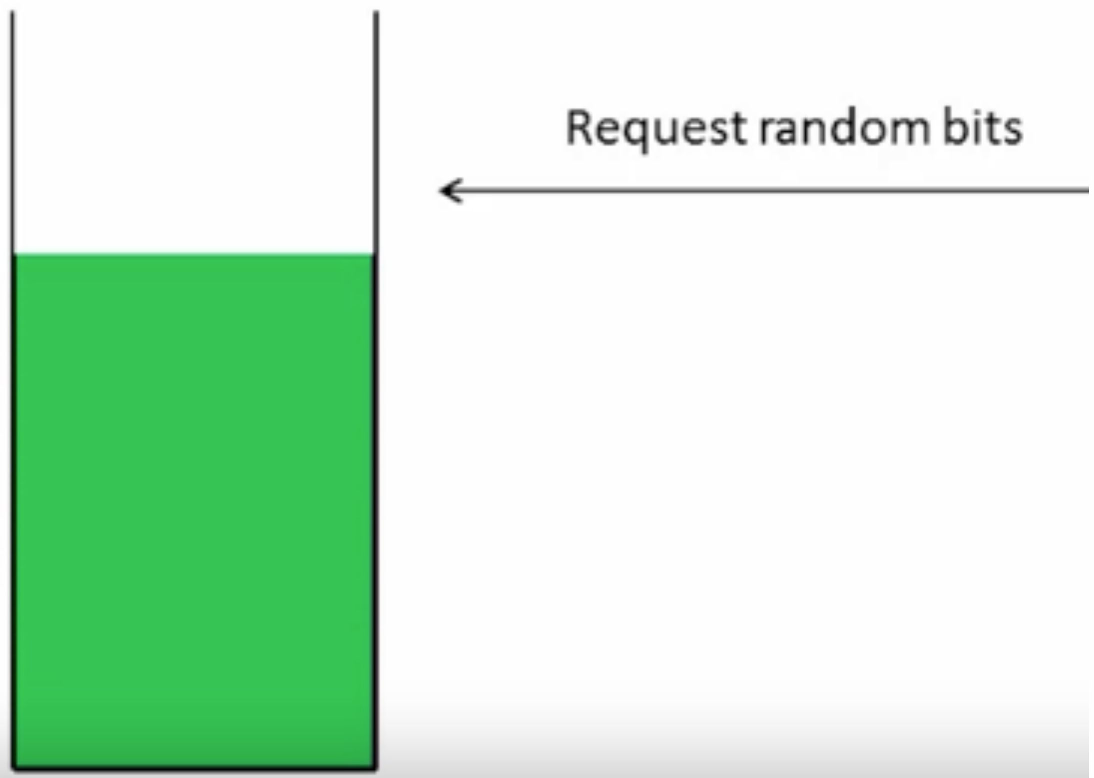
- Other… – Hardware random-number generator (e.g., Intel)

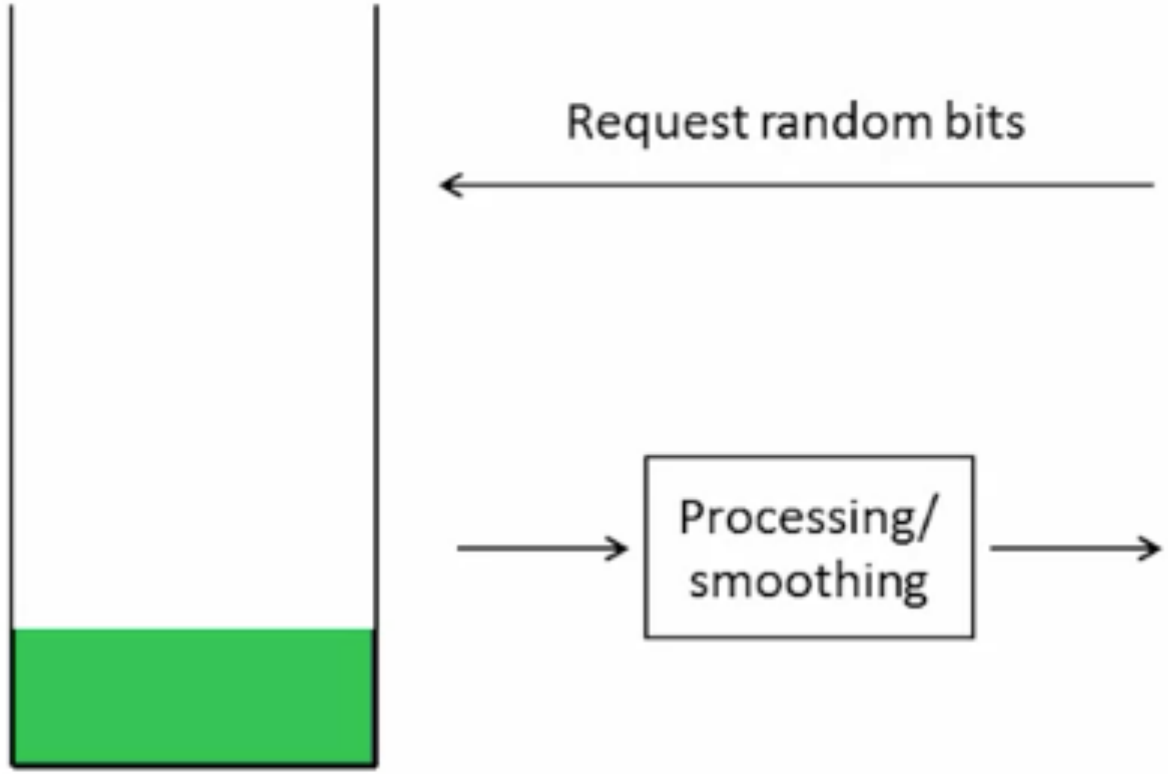
How might we get “good” random numbers?

- For security applications, want “cryptographically secure pseudorandom numbers”
- Libraries include cryptographically secure pseudorandom number generators (CSPRNG)

- Linux:
  - /dev/random: blocking: waits for enough entropy
  - /dev/urandom: nonblocking, possibly less entropy
  - getrandom() – syscall! – by default blocking
- Internally:
  - Entropy pool: gathered from multiple sources
    - e.g.: mouse/keyboard/network timings
- Better idea:
  - AMD/Intel’s on-chip random number generator: RDRAND
  - Hopefully no hardware bugs!

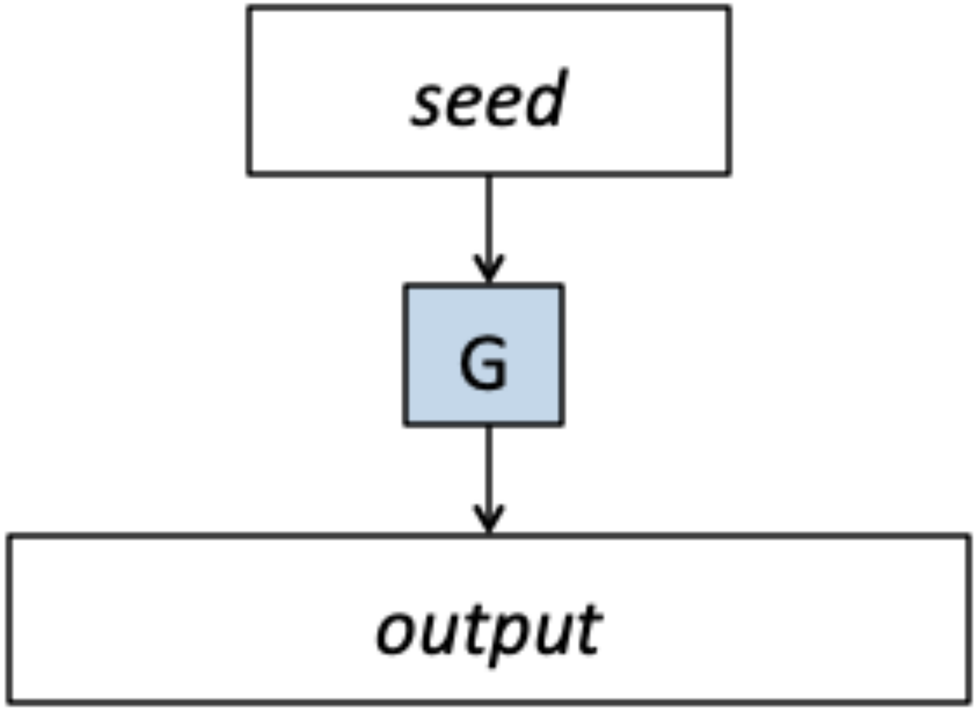
Random-number generation





Pseudorandom (number) generators: PRG/PRNGs

- A PRG is an efficient, deterministic algorithm that expands a short, uniform seed into a longer, pseudorandom output
  - Useful whenever you have a "small" number of true random bits, and want lots of "random looking" bits



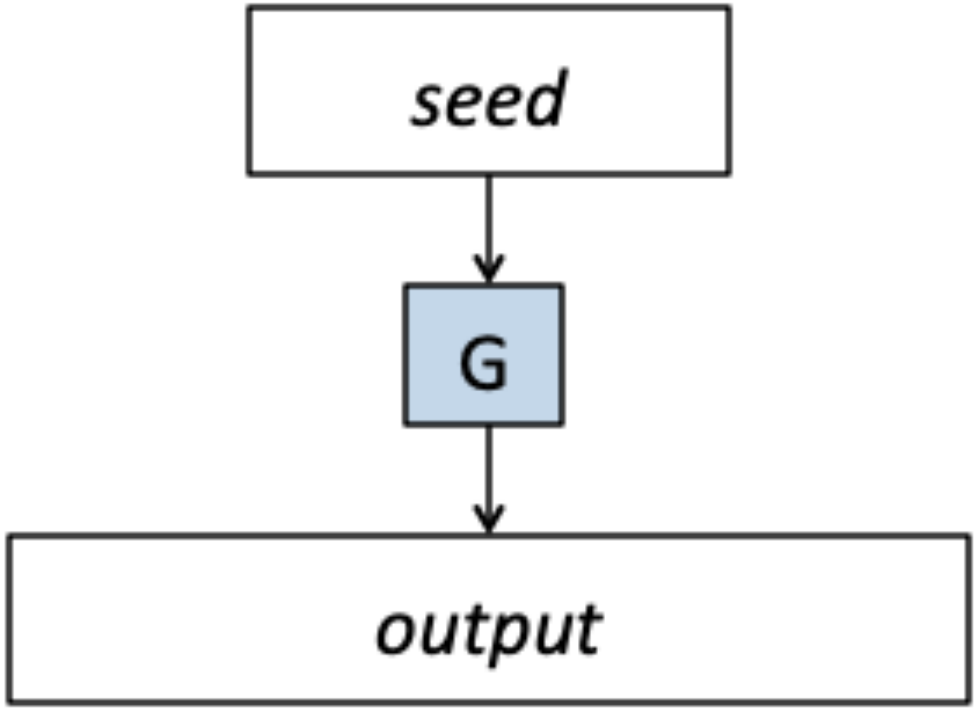
seed: a small number of true random bits

G: deterministic polynomial time algorithm

output: pseudorandom bits of length n, i.e., cannot be distinguished from truly random bits, by any efficient statistical test.

Do PRNGs exist?

- We actually don't know!
- Assume that there exist *some* functions G that are PRNG.



seed: a small number of true random bits

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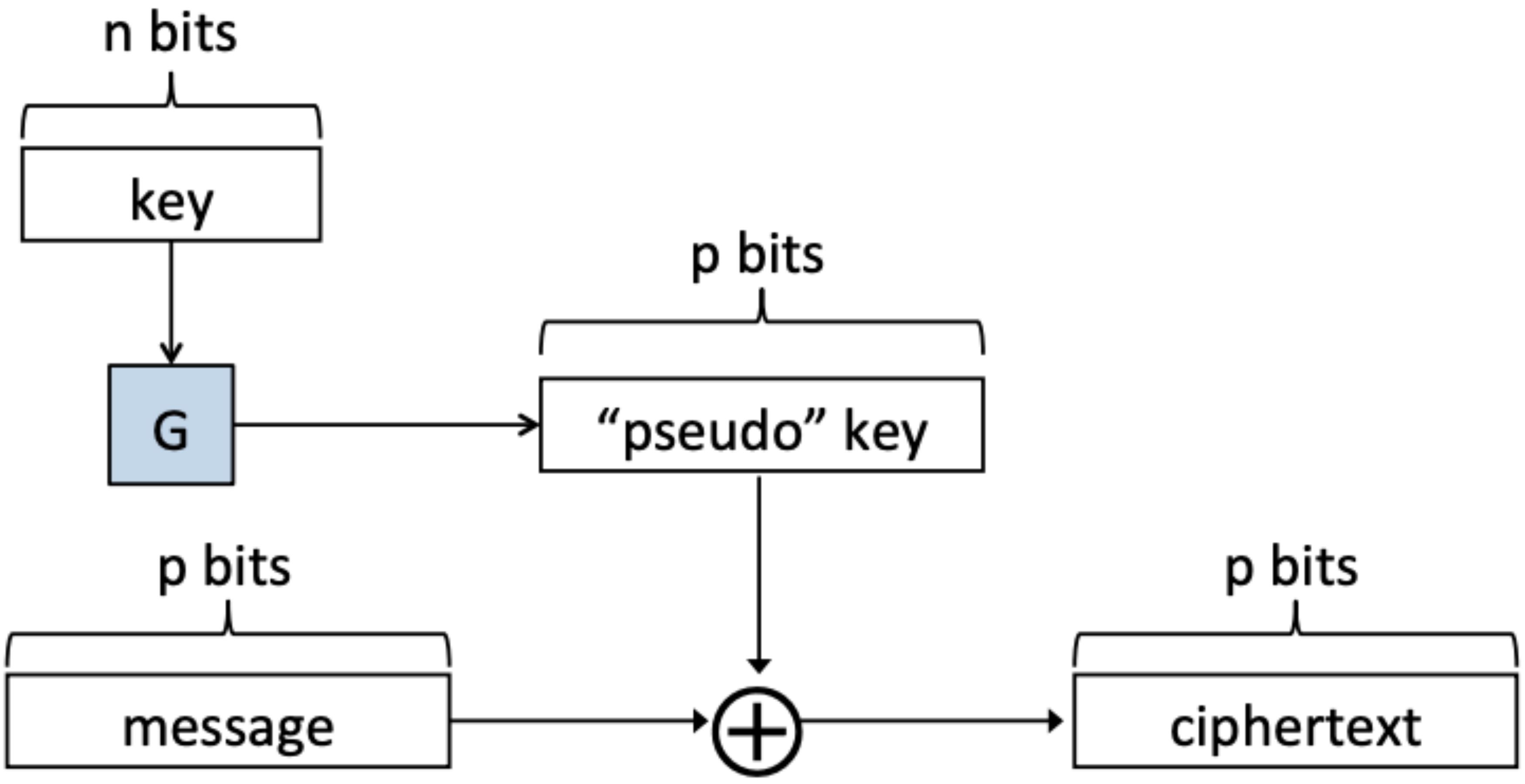
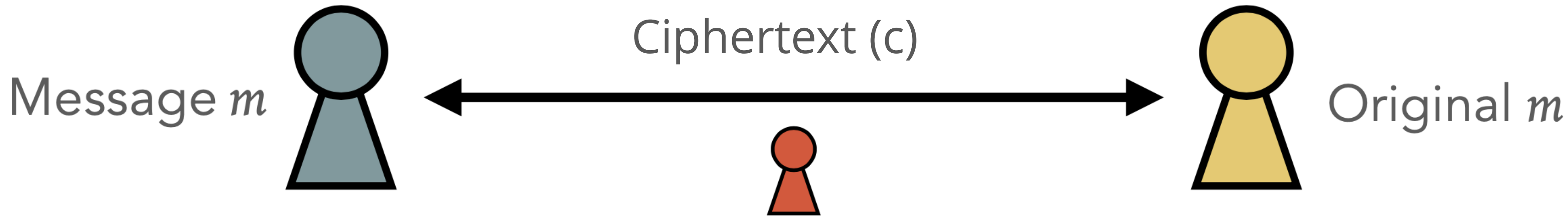
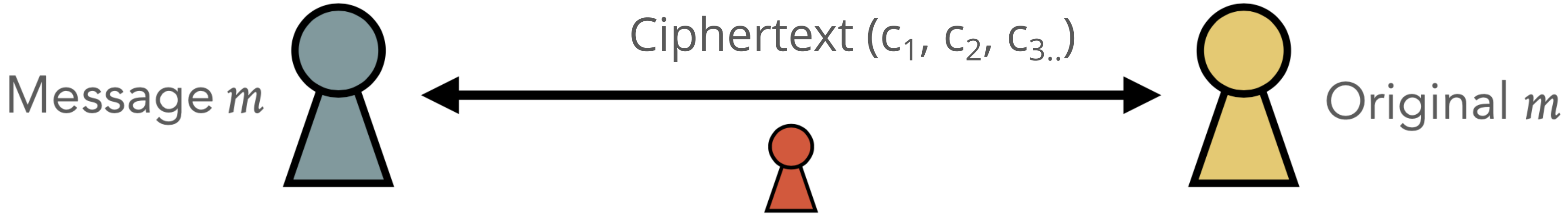
Applying Pseudo-randomness to the one-time pad



Single-message secrecy



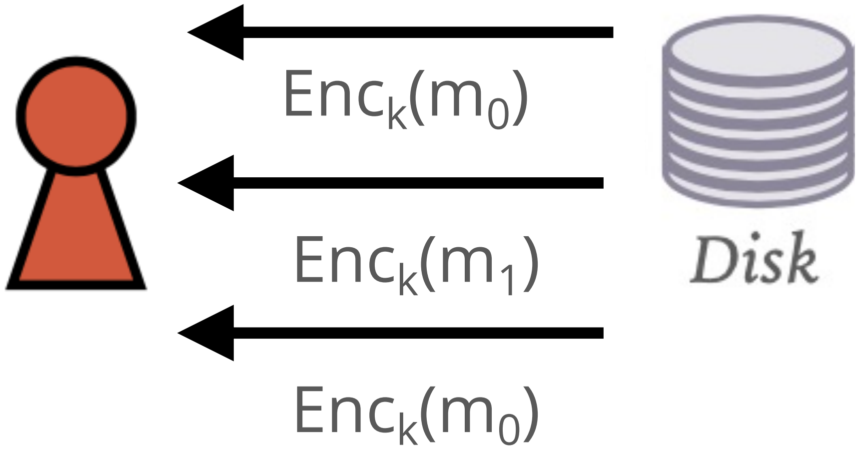
Multiple message secrecy

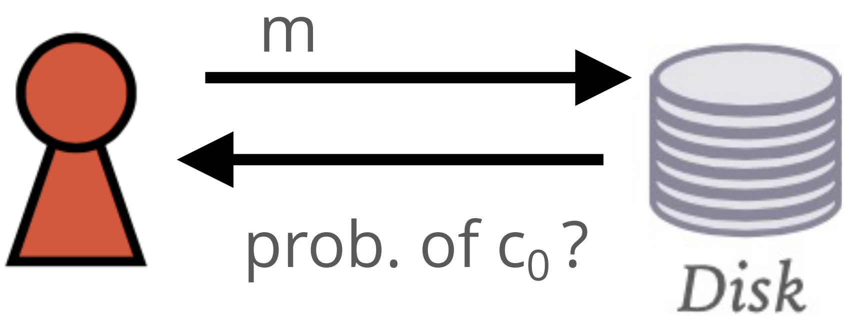


We are not going to formally define a notion of multiple-message secrecy

- Instead, define something stronger: security against chosen-plaintext attacks (CPA-security)
- *minimal notion of security an encryption scheme should satisfy*

Security against Chosen Plaintext Attack: Impossible?





It really is a problem if an attacker can tell when the same message is encrypted twice!

This attack only works if encryption is deterministic!

Random Functions

- Functions map from some set X to a set F(X) = Y.
  - (think of this mapping as a hash table mapping from x -> y)

- $Func_n$: all mappings from X: $\{0, 1\}^n$ -> F(X)= Y:$\{0, 1\}^n$
  - i.e., for all input bit strings of length n, there is a mapping to an output bit string also of length n
  - *all possible mappings? $2^{n.(2\^{}n)}$!! astronomically large!*

Random Functions

Out of all possible functions between X and Y we choose one uniformly at random.

- e.g. for a 2 bit string mappings between X: $\{0, 1\}^2$ and Y: $\{0, 1\}^2$
- one possible mapping that we could choose:

x	00	01	10	11
F(x)	01	11	00	10

Properties of function F(X) chosen uniformly at random:

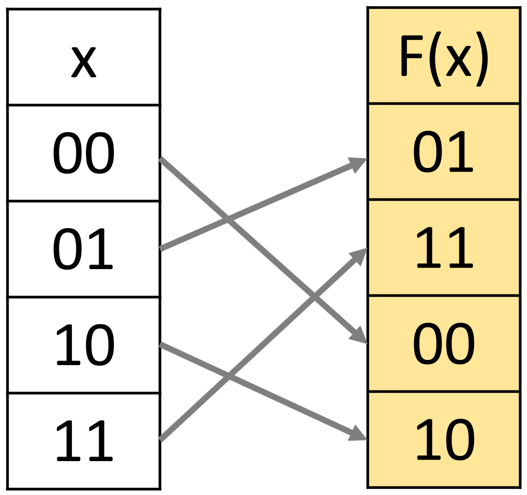
- for any given $x \in X$, the probability that $F(x) = y$ is $1/2^n$
- in our example example:
  - given $x \in X$, the probability that $F(x) = 1/2^2 = ¼ = 0.25$
- F(x) property:
  - if x changes by one bit to give x' then
  - *F(x') is completely independent of F(x).*

Random Permutations

- Variant of random function is random permutation
  - treat them equivalently for our purposes .

- E.g.: random permutation over bit strings of length 2

  Encryption: $\{0, 1\}^2 \rightarrow \{0, 1\}^2$



Important Property of the Random Permutation:

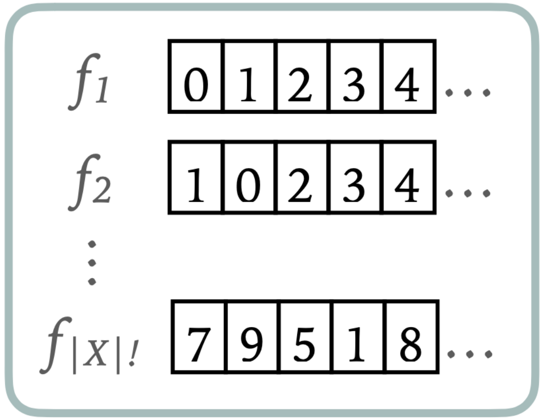
A permutation is invertible (bijective) function

Given F(x) it is impossible to determine x without resorting to a brute force attack.

If |X| is very large? brute force not possible by an efficient (probabilistic polynomial time) attacker.

What we have, ideally: Random Functions

Consider the set of all permutations $F_k \colon X \to X$



Think of X as all 128-bit bit strings

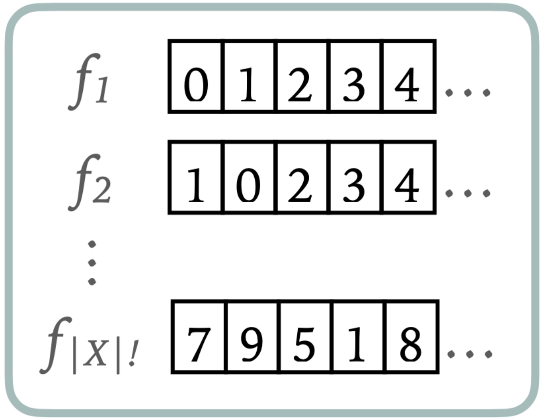
If you know k, then $F_k(x)$ is trivial to invert

If you don't know k, then $F_k(x)$ is one-way

<u>most efficient attack is a brute force attack.</u>

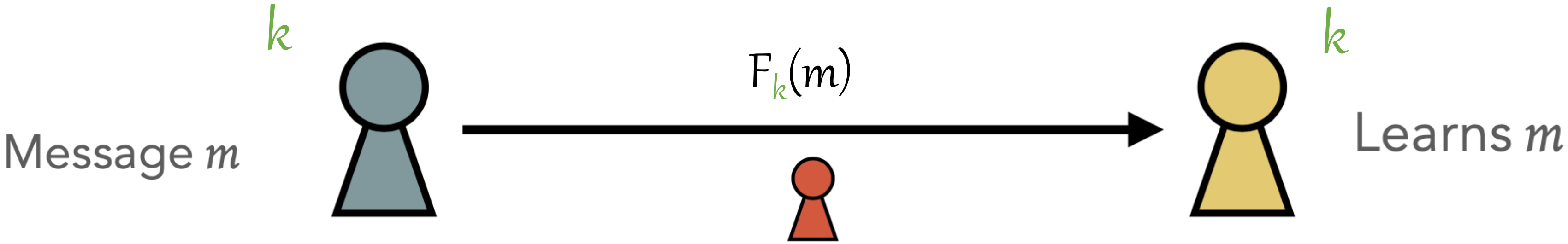
What we have, ideally: Random Functions

Consider the set of all permutations $F_k: X \to X$



Think of X as all 128-bit bit strings

Shared secret: index k chosen uniformly, at random



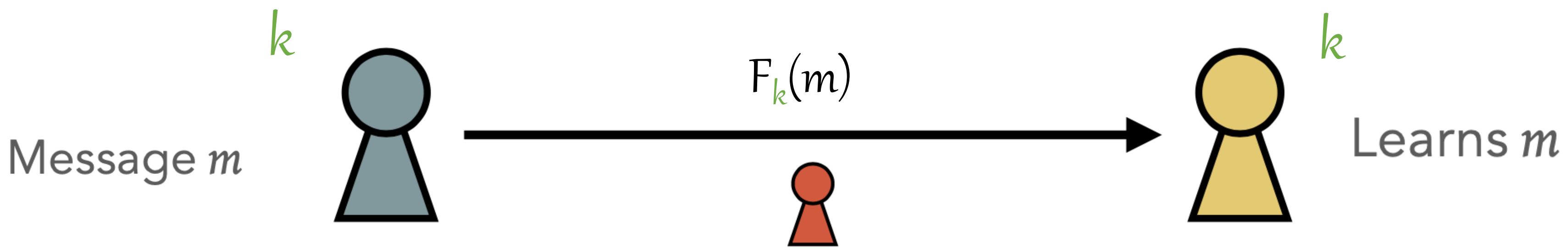
Without knowing k, Eve learns nothing about m

k is our key!

What we have, ideally: Random Functions

Shared secret: index k chosen uniformly, at random

k is our key!



Without knowing *k*, Eve learns nothing about *m*

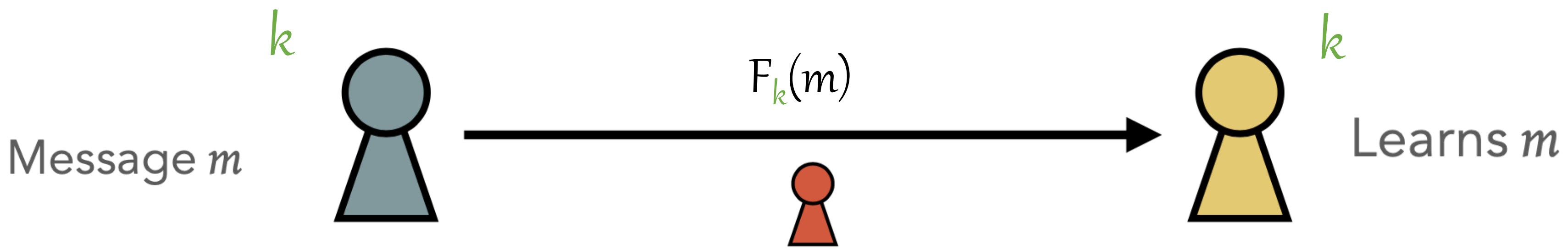
In essence, this protocol is saying “Let’s use the kth permutation function”

Infeasible to store all permutation functions – so instead cryptographers construct *pseudorandom functions*

What we have, approximately: Pseudo-Random Functions

Shared secret: index k chosen uniformly, at random

k is our key!



Without knowing *k*, Eve learns nothing about *m*

In essence, this protocol is saying "Let's use the kth permutation function"

Infeasible to store all permutation functions – so instead cryptographers construct *pseudorandom functions*