CS 31: Introduction to Computer Systems

02: Binary Representation

January 24

SWARTHMORE COLLEGE

Announcements

- Sign up for Piazza!
- Let me know about exam conflicts!
- Register your clicker!

Today

- Number systems and conversion
  - Decimal
  - Binary
  - Hexadecimal

- Data types and storage:
  - Data sizes
  - Representation

Reading Quiz

- Note the red border!
- 1 minute per question
- No talking, no laptops, phones during the quiz.

Abstraction

User / Programmer
Wants low complexity

Applications
Specific functionality

Software library
Reusable functionality

Operating system
Manage resources

Complex devices
Compute & I/O

Today

- Number systems and conversion

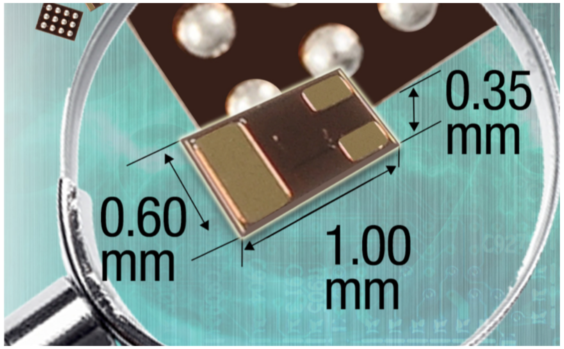
- Data types and storage:
  - Sizes
  - Representation
  - Signedness

Data Storage

- Lots of technologies out there:

Magnetic (hard drive, floppy disk)

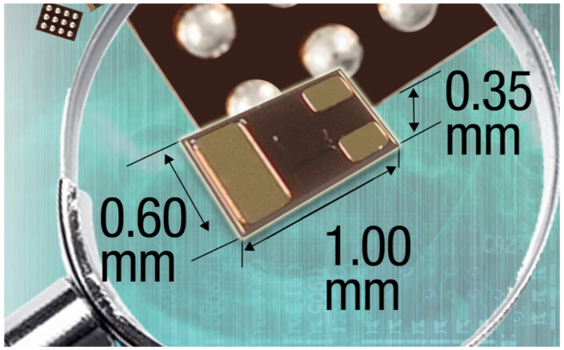
Optical (CD / DVD / Blu-Ray)



Electronic: RAM, registers

Electronic Data Storage

- Focus on electronic data storage



- Easy to differentiate two states
  - Voltage present
  - Voltage absent

We'll see (and build) digital circuits soon!

Binary Digits (Bits)

Bit: a 0 or 1 value (binary)

- Hardware represents as two different voltages
  - 1: the presence of voltage (high voltage)
  - 0: the absence of voltage (low voltage)

- Transistors: On or Off
- Optical: Light or No light
- Magnetic: Positive or Negative

Bits and Bytes

- Bit: a 0 or 1 value (binary)
  - HW represents as two different voltages
  - 1: the presence of voltage (high voltage)
  - 0: the absence of voltage (low voltage)

- Byte: 8 bits, the smallest addressable unit

  Memory: 01010101 10101010 00001111 …

- Other names:
  - 4 bits: Nibble
  - "Word": Depends on system, often 4 bytes (32 bits)

Files

Sequence of bytes… nothing more, nothing less

Binary Digits: (BITs)

- One bit: two values (0 or 1)
- Two bits: four values (00, 01, 10, or 11)
- Three bits: eight values (000, 001, …, 110, 111)

Discussion question

- Green border

- Recall the sequence
  - Answer individually (room quiet)
  - Discuss in your group (room loud)
  - Answer as a group
  - Class-wide discussion

How many unique values can we represent with 9 bits? Why?

- One bit: two values (0 or 1)
- Two bits: four values (00, 01, 10, or 11)
- Three bits: eight values (000, 001, …, 110, 111)

A. 18
B. 81
C. 256
D. 512
E. Some other number of values.

How many values?

1 bit: 0 1

2 bits: 0 0 0 1 1 0 1 1

3 bits: 0 0 0 0 0 1 0 1 0 0 1 1 1 0 0 1 0 1 1 1 0 1 1 1

4 bits: 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 1 16 values

0 1 0 0 0 1 0 1 0 1 1 0 0 1 1 1

1 0 0 0 1 0 0 1 1 0 1 0 1 0 1 1

1 1 0 0 1 1 0 1 1 1 1 0 1 1 1 1

N bits: 2^N values

Let's start with what we know

- Digits 0-9
- <u>Positional numbering</u>
- Digits are composed to make larger numbers
- Known as the <u>Base 10</u> representation

Decimal number system (Base 10)

Sequence of digits in range [0, 9]

6 4 0 2 5

Digit #0: 1's place (least significant digit)

Digit #1: 10's place

⋮

And so on

What is the significance of the N^{th} digit number in this number system? What does it contribute to the overall value?

64025

Digit #4: d_4 Digit #0: d_0

A. $d_N * 1$
B. $d_N * 10$
C. $d_N * 10^N$
D. $d_N * N^{10}$
E. $d_N * 10^{d_N}$

What is the significance of the N^{th} digit number in this number system? What does it contribute to the overall value?

64025

Digit #4: d_4 Digit #0: d_0

A. $d_N * 1$
B. $d_N * 10$
C. $d_N * 10^N$
D. $d_N * N^{10}$
E. $d_N * 10^{d_N}$

Consider the meaning of d_3 (the value 4) above. What is it contributing to the total value?

Decimal: Base 10

Favored by humans…

- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$
- where d is in {0,1,2,3,4,5,6,7,8,9},
- represents the value:

$$[d_n * 10^n] + [d_{n-1} * 10^{n-1}] + \ldots + [d_2 * 10^2] + [d_1 * 10^1] + [d_0 * 10^0]$$

64025 =

$6 * 10^4$	+	$4 * 10^3$	+	$0 * 10^2$	+	$2 * 10^1$	+	$5 * 10^0$
60000	+	4000	+	0	+	20	+	5

Generalizing

- The meaning of a digit depends on its position in a number.
- A number, written as the sequence of digits
  $d_n d_{n-1} \ldots d_2 d_1 d_0$ in base b represents the value:

$$[d_n * b^n] + [d_{n-1} * b^{n-1}] + \ldots + [d_2 * b^2] + [d_1 * b^1] + [d_0 * b^0]$$

Binary: Base 2

- Used by computers to store digital values.
- Indicated by prefixing number with **0b**
- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1}, represents the value:

$$[d_n * 2^n] + [d_{n-1} * 2^{n-1}] + \ldots + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

What is the value of 0b110101 in decimal?

- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1}, represents the value:

$$[d_n * 2^n] + [d_{n-1} * 2^{n-1}] + \ldots + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

A. 26
B. 53
C. 61
D. 106
E. 128

What is the value of 0b110101 in decimal?

- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1}, represents the value:

$$[d_n * 2^n] + [d_{n-1} * 2^{n-1}] + \ldots + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

A. 26
B. <u>53</u>
C. 61
D. 106
E. 128

Binary Digits: (BITS)

7 6 5 4 3 2 1 0

Most significant bit → <u>1</u>000111<u>1</u> ← Least significant bit

Representation: $1 \times 2^7 + 0 \times 2^6 \ldots\ldots + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

10001111 = 143

Other (common) number systems.

- Base 10: decimal
- Base 2: binary

- Base 16: hexadecimal
- Base 8: octal
- Base 64

Hexadecimal: Base 16

- Indicated by prefixing number with **0x**

- A number, written as the sequence of digits
  $d_n d_{n-1} \ldots d_2 d_1 d_0$
  where d is in {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F},

represents the value:

$$[d_n * 16^n] + [d_{n-1} * 16^{n-1}] + \ldots + [d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$$

What is the value of 0x1B7 in decimal?

A. 397
B. 409
C. 419
D. 437
E. 439

$$[d_n * 16^n] + [d_{n-1} * 16^{n-1}] + \ldots +$$

$$[d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$$

$16^2 = 256$

DEC	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HEX	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

What is the value of 0x1B7 in decimal?

A. 397
B. 409
C. 419
D. 437
E. <u>439</u>

$[d_n * 16^n] + [d_{n-1} * 16^{n-1}] + \ldots +$

$[d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$

$16^2 = 256$

$1*16^2 + 11*16^1 + 7*16^0 = 439$

DEC	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HEX	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Important Point…

- You can represent the same value in a variety of number systems / bases.

- It's **all** stored as binary in the computer.
  - Presence/absence of voltage.

Other (common) number systems.

- Base 2: How data is stored in hardware.
- Base 10: Preferred by people.
- Base 8: Used to represent file permissions.
- Base 16: Convenient for representing memory addresses.
- Base 64: Commonly used on the Internet, (e.g. email attachments).

<u>Different ways of visualizing the same information!</u>

Hexadecimal: Base 16

- Fewer digits to represent same value
  - Same amount of information!
- Like binary, base is power of 2
- Each digit is a “nibble”, or half a byte.

Each hex digit is a "nibble"

- One hex digit: 16 possible values (0-9, A-F)
- $16 = 2^4$, so each hex digit has exactly four bits worth of information.
- We can map each hex digit to a four-bit binary value. (helps for converting between bases)

Each hex digit is a "nibble"

Example value: 0x1B7

Four-bit value: 1

Four-bit value: B (decimal 11)

Four-bit value: 7

In binary:	0001	1011	0111
	1	B	7

Hexadecimal Representation

- Bit patterns as base-16 numbers
- Convert binary to hexadecimal: by splitting into groups of 4 bits each.

Example:

$$11\ 1100\ 1010\ 1101\ 1011\ 0011_2 = 3CADB3_{16}$$

Bin	11	1100	1010	1101	1011	0011
Hex	3	C	A	D	B	3

Converting Decimal -> Binary

- Two methods:
  - division by two remainder
  - powers of two and subtraction

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
  if D is odd
      set b_i to 1
  if D is even
      set b_i to 0
  i++
  D = D/2
```

Example: Converting 105

```
idea:   D = b          example: D = 105          a0 = 1
```

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
  if D is odd
      set b_i to 1
  if D is even
      set b_i to 0
  i++
  D = D/2
```

Example: Converting 105

```
idea:     D = b          example: D = 105          a0 = 1
          D/2 = b/2                D =  52          a1 = 0
```

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
  if D is odd
      set b_i to 1
  if D is even
      set b_i to 0
  i++
  D = D/2
```

Example: Converting 105

```
idea:     D = b          example: D = 105          a0 = 1
        D/2 = b/2                 D =  52          a1 = 0
        D/2 = b/2                 D =  26          a2 = 0
        D/2 = b/2                 D =  13          a3 = 1
        D/2 = b/2                 D =   6          a4 = 0
        D/2 = b/2                 D =   3          a5 = 1
          0 =  0                  D =   1          a6 = 1
                                  D =   0          a7 = 0

                                        105   =   01101001
```

Method 2

- $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$
- To convert 105:
  - Find largest power of two that's less than 105 (64)
  - Subtract 64 (105 – 64 = 41), put a 1 in d_6
  - Subtract 32 (41 – 32 = 9), put a 1 in d_5
  - Skip 16, it's larger than 9, put a 0 in d_4
  - Subtract 8 (9 – 8 = 1), put a 1 in d_3
  - Skip 4 and 2, put a 0 in d_2 and d_1
  - Subtract 1 (1 – 1 = 0), put a 1 in d_0 (Done)

1	1	0	1	0	0	1
d_6	d_5	d_4	d_3	d_2	d_1	d_0

What is the value of 357 in binary?

A. 101100011
B. 101100101
C. 101101001
D. 101110101
E. 110100101

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$,
$2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$,
$2^8 = 256$

What is the value of 357 in binary?

A. 101100011
B. 101100101
C. 101101001
D. 101110101
E. 110100101

$357 - 256 = 101$
$101 - 64 = 37$
$37 - 32 = 5$
$5 - 4 = 1$

1	0	1	1	0	0	1	0	1
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$,
$2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$,
$2^8 = 256$

So far: Unsigned Integers

- With N bits, can represent values: 0 to 2^n-1
- We can always add 0's to the front of a number without changing it:

10110= 010110 = 00010110 = 0000010110

- `1 byte: char, unsigned char`
- `2 bytes: short, unsigned short`
- `4 bytes: int, unsigned int, float`
- `8 bytes: long long, unsigned long long, double`
- `4 or 8 bytes: long, unsigned long`

Representing Signed Values

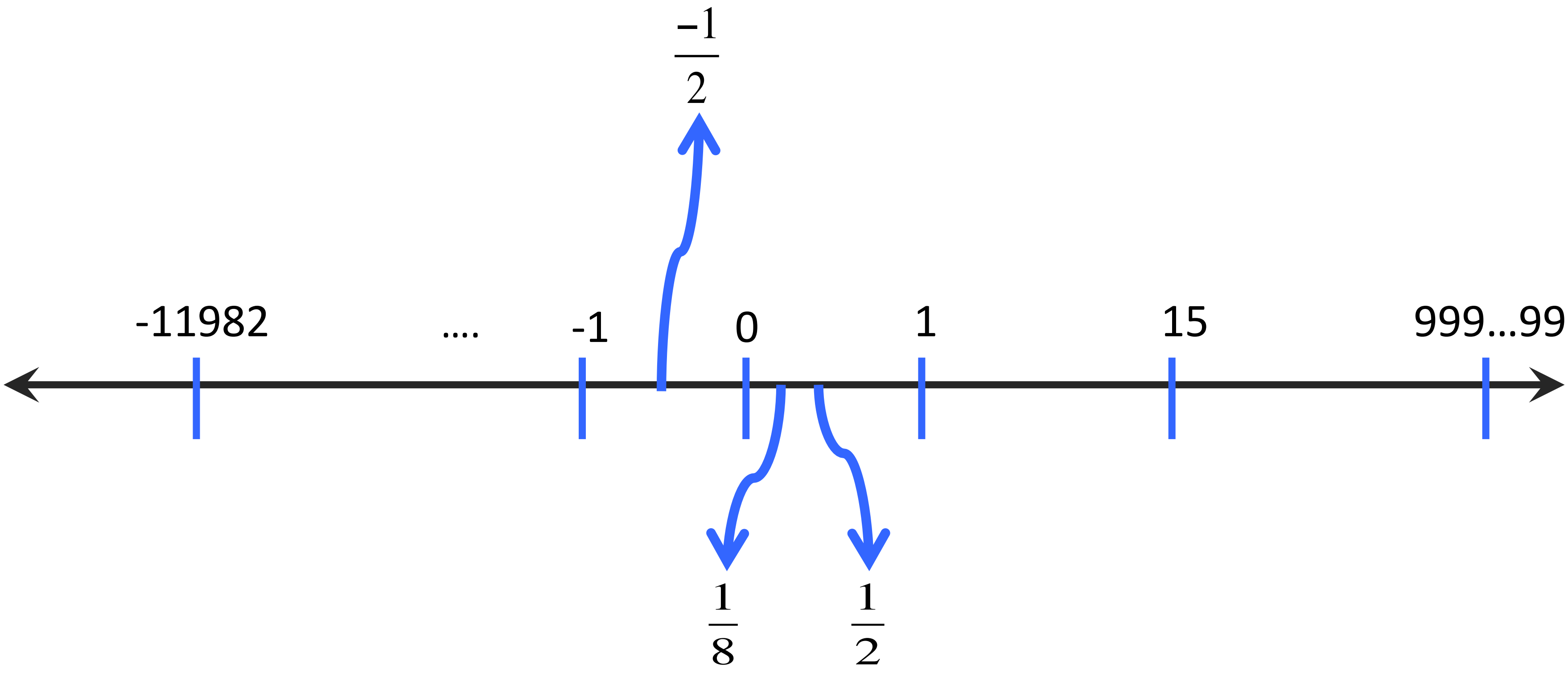
- One option (used for floats, <u>NOT integers</u>)
  - Let the first bit represent the sign
  - 0 means positive
  - 1 means negative

- For example:
  - <u>0</u>101 -> 5
  - <u>1</u>101 -> -5

- Problem with this scheme?

Fractional binary numbers

How do we represent fractions in binary?



Floating Point Representation

1 bit for sign   sign | exponent | fraction |
8 bits for exponent
23 bits for precision

$$\text{value} = (-1)^{\text{sign}} * 1.\text{fraction} * 2^{(\text{exponent}-127)}$$

let's just plug in some values and try it out

```
0x40ac49ba: 0 10000001   01011000100100110111010
      sign = 0 exp = 129   fraction = 2902458

               = 1*1.2902458*2^2 = 5.16098
```

<u>I don't expect you to memorize this</u>

Up Next: Binary Arithmetic