Gaming Prediction Markets: Equilibrium Strategies with a Market Maker

Main Presentation
A historical note...

*Non-myopic Strategies in Prediction Markets*
Dimitrov & Sami

*Bluffing and Strategic Reticence in Prediction Markets*
Chen, Reeves, Pennock, Hanson, Fortnow, & Gonen

=Gaming Prediction Markets: Equilibrium Strategies with a Market Maker*
Chen, Dimitrov, Sami, Reeves, Pennock, Hanson, Fortnow, & Gonen
Are prediction markets manipulable?

- Truthful betting is myopically optimal.

- Is truthful betting strategically optimal?

- Answer: it depends.
Information Structure

- Unconditionally independent
  
  $s_i \rightarrow w \rightarrow s_j$

- Conditionally independent
  
  $s_i \rightarrow w \rightarrow s_j$

- Conditionally independent
  
  $s_i \rightarrow w \leftarrow s_j$
Substitutable Signals

Each additional signal is less informative than the last.

Quality of materials example.

- \( \text{pr(break)} \in \{0.1, 0.01\} \)
- signal: did your product break?

If you already know the signals from 100 customers, you have a good estimate of \( \text{pr(break)} \); you don’t get much information from another signal.
Complementary Signals

Signals are more informative in combination than separately.

Sports Example:

- signal: the team’s prior record

It’s helpful to know one team’s record, but much more helpful to know both teams’ records.
Alice - Bob Game

- If the agents get only one chance to move, they should behave myopically.

- This means that on their last chance to move agents should be truthful.

- Whether you prefer to go first or second depends on signal structure:

  \[ CI \Rightarrow \begin{array}{c} \text{prefer } 1^{\text{st}} \\ (\text{substitutes}) \end{array} \quad w \quad \begin{array}{c} s_i \\ s_j \end{array} \quad I \Rightarrow \begin{array}{c} \text{prefer } 2^{\text{nd}} \\ (\text{complements}) \end{array} \quad s_i \quad s_j \quad w \]
Alice - Bob - Alice Game

- The last two moves will be truthful.
- Will Alice bluff on the first move?
- A bluff is defined as any move other than full revelation.

Again, it depends on signal structure:

\[ CI \Rightarrow \text{no bluff} \]
\( (\text{race to reveal}) \)

\[ l \Rightarrow \text{bluff} \]
\( (\text{wait it out}) \)
Infinite-Stage Game

- This is a more-realistic model: it’s possible to trade in a PM lots of times.

- Under CI, you still want to reveal your information as fast as possible.

- Under I, you still want to delay revealing your information.
Discounted LMSR

- Log score is discounted by $\delta^t$.
  - $\delta < 1$ is a discount rate.
  - $t$ is the number of previous trades.
- Encourages earlier trading.
- The equilibrium of the CI game is unchanged.
- The equilibrium of the I game is still not known, but the authors put a lower bound the amount of information revealed by time $t$.
- The bound is increasing in $t$ such that in the limit all information is revealed.
Discussion Questions

- What is the equilibrium of the I-game?
- Is it really bluffing if they just wait?
- Are signals ever actually I or CI?
- What does this mean for implementing PMs?