
Internet Advertising and the Generalized Second Price Auction: Selling Billions of Dollars Worth of Keywords

Main Presentation
by
Daniel May

Why not VCG?

- Generalized Second Price Auction
 - No equilibrium in dominant strategies
 - Typically equilibrium is not truthful
 - Easier to compute and explain to advertisers
 - Higher revenues than VCG



Static GSP Assumptions

- Bids change quickly and continuously
 - Could be complex equilibria, but assume there aren't
- Assume bids form equilibrium in static one-shot game with **complete information**
 - All values are common knowledge
 - Stable bids are static best response to other players'

Static GSP Equilibrium

- Player can force out player immediately above
 - Player in slot i below can increase the Player in slot $i+1$ payment enough so that Player in $i+1$ lowers his bid, and positions are swapped
- Eventually come to “locally envy-free” rest point

Locally Envy-Free Equilibrium

- Player will not want to swap with player immediately above or below
 - Paper proves it is a stable matching between advertisers and ad slots
- Lemma 5: Locally envy-free equilibrium of an auction is a stable assignment
- Lemma 6: If # bidders $>$ # slots, any stable assignment is locally envy-free equilibrium of the auction

Locally Envy-Free Equilibrium (cont.)

- Equilibrium bidders' payments are the same as payments in dominant-strategy equilibrium of VCG
- Equilibrium is the worst locally envy-free equilibrium for the search engine and best locally envy-free equilibrium for the bidders

Theorem 7

- Shows that the payments locally envy-free is the same as VCG
- Shows that bidder cannot benefit by bidding less than equilibrium bid
 - Non-truthful bidding is not profitable in VCG, not profitable here
- Show that the equilibrium revenue is best possible locally envy-free outcome for bidders and worst possible outcome for the search engine

Significance of Theorem 7

- Locally envy-free obtains outcome similar to dominant-strategy equilibrium of the game induced by VCG
 - Advertisers select the position that makes them locally envy-free
 - Search engine gets \$\$\$ \geq VCG with an easy computation method
 - Not best locally envy-free equilibrium, but at least as good as VCG

Generalized English Auction

- Clock with a price increases over time
 - Player's bid is price when they drop out
- Auction ends when next-to-last advertisers drops
- Very myopic procedure



Why analyze Generalized English Auction?

- Static GSP assumes long-run steady state
 - Can get there by starting bid at 0 and incrementally increasing it
 - Generalized English Auction
- Generalized English Auction has same equilibrium as VCG, which is worst-case for GSP
 - Equilibrium are roughly equivalent
- Bids get used to calculate prices using GSP
- Easier to analyze and prove things

Theorem 8 Notation

- α_k - click-rate at slot k
- s_i - value of click to bidder
- b_k - bid price per click
- p - drop out price

Theorem 8

- $\alpha_k (s_i - b_{k+1})$
 - profit for slot k
- $\alpha_{k-1} (s_i - p)$
 - profit for slot k - 1
- Prove that s_i is the optimal drop out point for agent i
 - If wait, could get slot k and gain nothing get slot k-1, at higher price and preferred k
 - If drop out before your value, miss opportunity to get k-1 at a cheaper price

Significance of Theorem 8

- Dominant strategies do not exist
- Generalized English auction payoffs coincide with VCG payments for all realizations of values
 - Bidders can be asymmetric
 - Distributions of values need not be known
- Unique and efficient equilibrium exist, but bidders do not have dominant strategies
 - Has ex post equilibrium

Conclusion

- Generalized Second Price auctions perform as well as VCG
- “Emerged in the wild,” but this paper proved its worth



Questions?

