Motivation

- To have an evolutionarily stable strategy (ESS), a population using only that strategy must not be able to be invaded by any other, initially rare, strategy.
- Every ESS is a Nash Equilibrium, but not the other way around.
Setting Up the Hawk/Dove Game

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<tr>
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<th>Hawk (A)</th>
<th>Dove (B)</th>
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<tr>
<td>Hawk (A)</td>
<td>$a$</td>
<td>$b$</td>
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<tr>
<td>Dove (B)</td>
<td>$c$</td>
<td>$d$</td>
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Assumptions

- **Graph is regular**
  - Every player interacts with the same number of players

- **Two-agent interactions**
  - When the two players interact, only those players’ strategies affect the outcome

- **Fixed spatial relationship**
  - Players only ever interact with the same players
Update Rules

- Sum the payoffs for interactions with all surrounding players separately for each strategy
- The strategy with the greater total payoff is more likely to replace the other
Update Rules

\[(k - 1)a + b\]
Update Rules

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Update Rules

$$(k - 1)a + b$$
Well-Mixed Population
Well-Mixed Population

- The condition here for evolutionary stability is given by:
  \[(a - c)(1 - \varepsilon) + (b - d)\varepsilon > 0\]

- where \(\varepsilon\) is the fraction of B-strategists

- as \(\varepsilon\) goes to zero, the inequality simplifies to:
  \[a > c\]
Graph-Structured Population
Graph-Structured Population

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<td>$c \approx a$</td>
<td>$d &lt; b$</td>
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Graph-Structured Population
Graph-Structured Population

Half-line of $B$ invaders

extend? or shrink?

$k_a$

$(k-1)a+b$

$(k-1)c+d$

$(k-2)c+2d$
Simulations
Simulations

- Triangular, $L=6$
- Square, $L=4$
- Hexagonal, $L=3$
Related Reading

- Killingback and Doebeli, 1996, *Spatial Evolutionary Game Theory: Hawks and Doves Revisited*

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<td>1 - β</td>
<td>2</td>
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<tr>
<td>Dove  (B)</td>
<td>0</td>
<td>1</td>
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Related Reading
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Discussion Questions

- In what applications might the warning about lattices be pertinent? Where do we see cyclical relationships in game theory applications?
- What kinds of dynamics might we expect when we break the earlier assumptions? For example, what might this look like on a non-regular graph?
- Other than theoretical biology, what are the applications of the evolutionary stability concept? In what scenarios would knowledge of an ESS be helpful?