

EVOLUTIONARY STABILITY ON GRAPHS

Background Presentation
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Evolutionary Game Theory

- Evolutionary game theory
 - ▣ Applies game theory to model the way that populations change over time
 - ▣ This makes modeling animal's survival strategies easier
- Evolutionarily stable strategy (ESS)
 - ▣ A mutant strategy will not be able to invade if every member of a population adopts an evolutionarily stable strategy
 - ▣ ESS's become the equilibrium states and this is what is interesting to study

Well mixed population vs. Realistic population

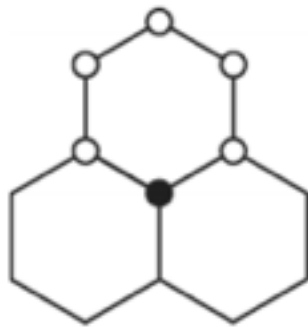
- Well mixed population
 - ▣ In the past ESS have been adopted for a well mixed population that is infinite
 - ▣ In other words every one is equally likely to interact
- Realistic population
 - ▣ Realistically, a member of a population is more likely to interact more with its neighbors than with someone who is very far away
 - ▣ Also the population is not infinite

Spatial Evolutionary Game Theory

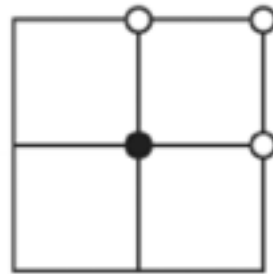
- Members of a population are distributed on a evenly spaced grid
- Each member of the population only interacts with its neighbors
- Strategies interact with neighboring regions and are updated if the neighboring strategies work better
- Spatial games behave much differently than those of well-mixed populations but are more realistic

Evolutionary Graph Theory

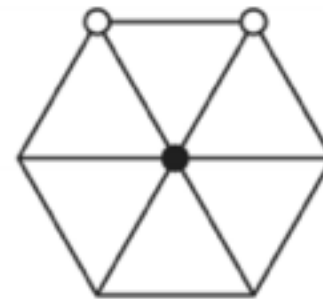
- In evolutionary graph theory populations are represented as graphs
- Spatial games can be represented as lattices
- Well mixed can be represented by evenly weighted complete graphs



triangular, $L=6$



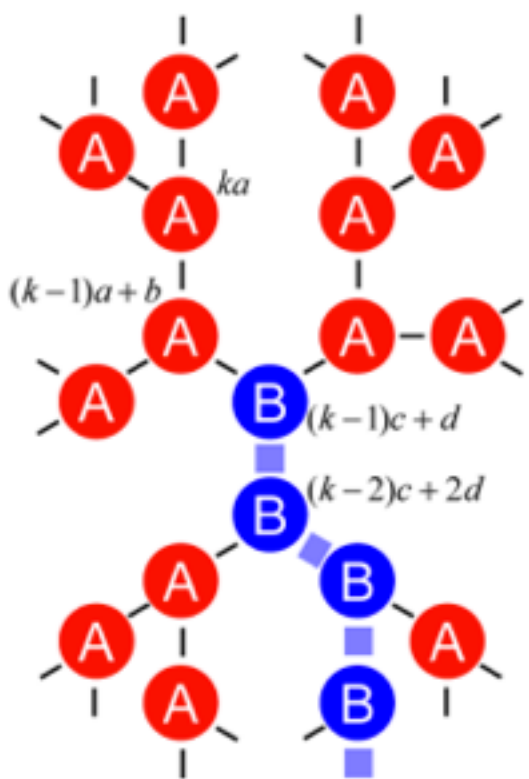
square, $L=4$



hexagonal, $L=3$

Evolutionary Stability for Well-Mixed Population

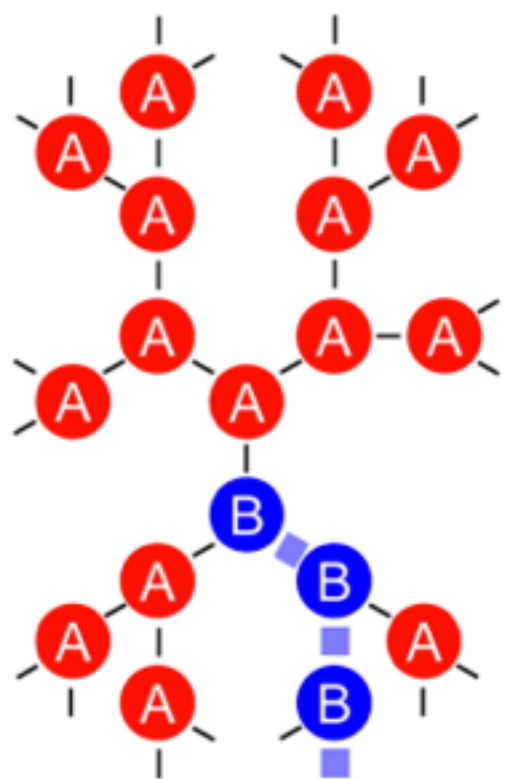
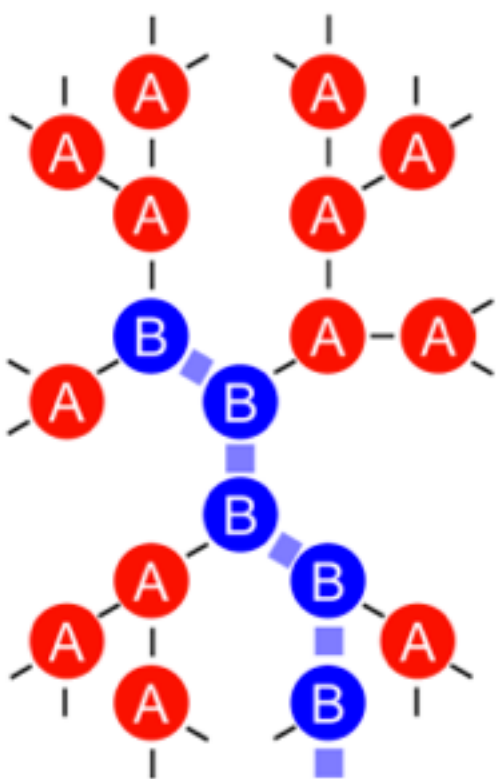
**Half-line of
B invaders**



extend?

or

shrink?



Update Rules

- Birth-Death (BD)
 - ▣ An individual is chosen for reproduction proportional to its fitness and will replace one of its neighbors who is chosen randomly
- Death-Birth (DB)
 - ▣ An individual is randomly chosen to die and its neighbors fight for its place proportional to their fitness
- Imitation (IM)
 - ▣ An individual is randomly chosen to update his strategy and either sticks with it or changes to one of his neighbor's strategies proportional to fitness

Other Reading

- Killingback, Timothy, and Michael Doebeli. "Spatial evolutionary game theory: Hawks and Doves revisited." *Proceedings of the Royal Society of London B: Biological Sciences* 263.1374 (1996): 1135-1144.



Questions?