CS91 Problem Set 4

This problem set is due at 11:59pm on Sunday, November 18th. Submit this problem set using github. For this problem set, you will work with a partner. It’s ok to discuss approaches with others at a high level, but most of your discussions should just be with your partner, and the write-up must your (and your partner’s) own original work.

Problem 1

For the weighted majority algorithm, we proved the following bound on \( M \) (the number of mistakes made by the algorithm) in terms of \( OPT \) (the number of mistakes made by the best expert):

\[
M \leq \log_{4/3}(N) + OPT \cdot \log_{4/3}(2)
\]

Consider an alternative algorithm called “follow-the-leader”, that at each round considers only the advice of the experts who have made the fewest mistakes so far. Among those experts, it makes the majority prediction, breaking ties in favor of \( D \).

(A) Describe an example where follow-the-leader makes more than \( \log_{4/3}(N) + OPT \cdot \log_{4/3}(2) \) mistakes. **Hint:** your example will need many rounds; for \( N = 10 \) and \( OPT = 1 \), the bound comes out to impossibly many (\( \approx 10.4 \)) mistakes. It may also help to have lots of experts.

Solution

(B) How many mistakes does the weighted majority algorithm make on your example?

Solution

Problem 2

The minimax theorem (as we stated it) says that in any two-player zero-sum game the players have opposite safety values: \( SV_1 = -SV_2 \). For this problem, you will show several corollaries that follow from the minimax theorem.

(A) Prove that all two-player zero-sum games have at least one Nash equilibrium.

Solution

(B) Prove that in a two-player zero-sum game, all Nash equilibria have the same value. That is, \( u_1(NE) = u_1(NE') \) for any two equilibria \( NE \) and \( NE' \).

Solution
(C) Prove that in a two-player zero-sum game, all Nash equilibrium strategies are interchangeable. That is if \((\sigma_1, \sigma_2)\) and \((\sigma'_1, \sigma'_2)\) are Nash equilibria, then so are \((\sigma'_1, \sigma_2)\) and \((\sigma_1, \sigma'_2)\).

Solution