CS91 Problem Set 2

This problem set is due at **11:59pm on Sunday, October 28th**. Submit this problem set using [github](https://github.com). For this problem set, you will work with a partner. It’s ok to discuss approaches with others at a high level, but most of your discussions should just be with your partner, and the write-up must your (and your partner’s) own original work.

Problem 1 (based on TLAGT exercise 7.4):

In auctions that happen online, for example on Ebay, it is often possible for a bidder to create multiple accounts in order to submit several bids to the same auction, unbeknownst to the mechanism. In this case, we need to model a bidder’s allocation as the union of all goods their accounts receive, and their payment as the sum of all payments their accounts are charged.

(A) Prove that in a second-price single-item auction, this never helps. In other words, even if users have multiple accounts, the dominant strategy is to bid truthfully on one account and bid zero on all other accounts.

Solution

(B) Prove that in a VCG mechanism, it is possible for a bidder to earn higher utility by submitting bids through multiple accounts than by bidding truthfully on a single account. You should assume in your counterexample that all other bidders use one account and bid truthfully.

Solution

Problem 2 (based on TLAGT exercise 9.5):

An alternative mechanism for the gift exchange problem is called *random serial dictatorship*. You may already be familiar with this mechanism, because it is used to run the Swarthmore housing lottery.

```
V ← {all items}
randomly order the agents
for i = 1...n do
    v ← agent i’s favorite item in V
    V ← X \ {v}
end for
```

(A) Is this mechanism DSIC? Prove or give a counterexample.

Solution
Problem 3 (related to TLAGT exercise 10.7 and problem 10.1):

In class, we discussed a deferred-acceptance mechanism for a stable matching problem with the same number of doctors and hospitals: $|D| = |H|$. However in reality, there are many more new doctors than hospitals, and most hospitals can accept more than one medical resident.

(A) Extend the doctor-optimal deferred-acceptance mechanism to a setting where each hospital $h \in H$ can accept a number of residents $r_h$. As before, each doctor expresses a total ordering over the hospitals, and each hospital expresses a total ordering over the doctors.

(B) Prove that in your mechanism, it is a dominant strategy for every doctor to truthfully report their preferences over hospitals.

(C) Prove that in your mechanism, it is **not** a dominant strategy for every hospital to truthfully report its preferences over doctors.