CS91 Problem Set 2

This problem set is due at 11:59pm on Sunday, October 7th. Submit this problem set using github. For this problem set, you will work with a partner. It’s ok to discuss approaches with others at a high level, but most of your discussions should just be with your partner, and the write-up must your (and your partner’s) own original work.

Problem 1 (based on TLAGT problem 3.2):

Consider a the following non-ideal auction for the $k$-unit setting with unit-demand. The seller has a specific revenue target $R$, and tries to achieve it as follows:

$$
S ← \{\text{top } k \text{ bidders}\}
$$

while $\min_{i \in S} b_i < \frac{R}{|S|}$ do

remove the lowest bidder from $S$

end while

allocate an item to each remaining bidder in $S$ at price $\frac{R}{|S|}$

(A) Prove that the Revenue Target Auction is DSIC.

Solution

(B) Prove that whenever the $k+1^{st}$ price auction achieves revenue at least $R$, the Revenue Target Auction also achieves revenue at least $R$.

Solution

(C) Give an example of a valuation profile for which the $k+1^{st}$ price auction does not achieve revenue at least $R$, but the revenue target auction does.

Solution

Problem 2 (based on TLAGT problem 4.3):

Consider a set $M$ of distinct items. There are $n$ bidders, and each bidder $i$ has a publicly known subset $T_i \subseteq M$ of items that it wants, and a private valuation $v_i$ for getting exactly that subset. Since each item can only be awarded to one bidder, a subset $W$ of bidders can all receive their desired subsets simultaneously if and only if $T_i \cap T_j = \emptyset$ for each distinct $i, j \in W$.

(A) Show that this is a single-parameter environment.

Solution
Maximizing welfare in this setting is another example of an NP-hard problem, so we can’t expect to come up with an ideal auction. Instead, here is a greedy approximation algorithm for the social welfare maximization problem, given bids \( \tilde{b} \) from the bidders:

\[
W \leftarrow \emptyset \\
X \leftarrow M \\
\text{sort agents by bid so that } b_1 \geq b_2 \geq \ldots \geq b_n \\
\text{for } i = 1 \ldots n \text{ do} \\
\quad \text{if } T_i \subseteq X \text{ then} \\
\quad \quad W \leftarrow W \cup \{i\} \\
\quad \quad X \leftarrow X \setminus T_i \\
\quad \text{end if} \\
\text{end for} \\
\text{return winning bidders } W
\]

(B) Does this algorithm constitute a monotone allocation rule? Prove or give a counterexample.

Solution

(C) Prove that if all bidders report truthfully and want at most \( d \) items—\( |T_i| \leq d \forall i \)—then this allocation rule achieves social welfare at least \( \frac{1}{d} \) times that of the optimal allocation.

Solution

Problem 3 (based on TLAGT exercise 5.1):

Consider a second-price single-item auction with two bidders whose valuations are drawn independently from a uniform distribution \( v_i \sim U[0, 1] \).

(A) Prove that the expected revenue with no reserve price \( (R = 0) \) is \( \frac{1}{3} \).

Solution

(B) Give a formula for the expected revenue as a function of the reserve price \( R \).

Solution

(C) Find the revenue-maximizing reserve price and prove that it is optimal.

Solution