This problem set is due at **11:59pm on Sunday, September 23rd**. Submit this problem set using [github](https://github.com). For this problem set, you will work with a partner. It’s ok to discuss approaches with others at a high level, but most of your discussions should just be with your partner, and the write-up must your (and your partner’s) own original work.

**Problem 1** Is the generalized second-price auction dominant-strategy incentive compatible? Prove your answer.

**Solution**

**Problem 2** In lab 1, we proved\(^1\) that if a person’s preference relation \(\succsim\) obeys the five axioms, then there exists an expected utility representation of their decisions. In this problem, you will prove the reverse relationship: if a person is an expected utility maximizer, then there exists a preference relation satisfying all five axioms that represents their decisions.

(a) Describe the preference relation. That is, given a set of outcomes \(X\), and a utility for each outcome, describe how you would determine whether \(L_1 \succsim L_2\).

(b) Prove that the preference relation you have described obeys the completeness, transitivity, reduction, continuity, and independence axioms.

**Solution**

**Problem 3** In lab 2, we observed that the in the Nash equilibrium of the penalty shot game, the shooter was indifferent between KL and KR, while the goalkeeper was indifferent between JL and JR. Prove that this is true in general: in any mixed-strategy Nash equilibrium \(\vec{\sigma}\) of any game \(\Gamma\), for each player \(i\), and any pure strategy \(s\) that player \(i\) plays with non-zero probability, the expected utility of \(s\) and that of player \(i\)’s mixed strategy \(\sigma_i\) are equal: \(u_i(s, \vec{\sigma}_{-i}) = u_i(\sigma_i, \vec{\sigma}_{-i})\).

**Solution**

**Problem 4** In a first-price sealed-bid auction, there is no dominant strategy, and the equilibrium depends on the number of bidders and what they know about each other. Consider a slight modification to our in-class auction exercise: values drawn uniformly between 0 and 1: \(v_i \sim [0, 1]\).

(a) When there are \(n = 2\) bidders, prove that if both players bid \(\frac{v_i}{2}\), there is no other strategy that can increase either player’s expected utility.

(b) When there are \(n = 4\) bidders, prove that if all players bid \(\frac{3}{4}v_i\), there is no other strategy that can increase any player’s expected utility.

\(^1\)in the optional question 6
These strategies are examples of Bayes-Nash equilibria, an extension of Nash equilibrium to games where players have partial or probabilistic information about other players or the environment.