Mechanism Design
Without Money

Oct. 4, 2018
Simultaneous Ascending Auctions

• $k$ simultaneous single-item auctions

• Open-outcry “English” auctions (or similar)
  • Promotes price discovery

• Need an “activity rule” to prevent *sniping*. 
Demand Reduction

Suppose agent 1 has value-per-good of 10 for A,B:

\[ v_1(A) = v_1(B) = 10 \quad v_1(AB) = 20 \]

But agent 2 only wants one:

\[ v_2(A) = v_2(B) = v_2(AB) = 8 \]

How should agent 1 bid?

\[
\begin{align*}
\text{VCG} & \quad 1 \leftarrow AB \\
\text{SAA} & \quad 1 \leftarrow AB
\end{align*}
\]
Exposure Problem

Suppose goods A and B are complements for agent 1:
\[ v_1(A) = v_1(B) = 0 \quad v_1(AB) = 10 \]

But for agent 2 they are substitutes:
\[ v_2(A) = v_2(B) = v_2(AB) = 8 \]

How should agent 1 bid?
Package Bidding

We can combat the exposure problem by giving agents some capacity to bid on specific sets of goods.

Clearly, we need to limit the scope of package bids, otherwise we’re back to a fully combinatorial auction.

A possible approach: specify a few packages in advance.

What could go wrong here?
Problem: Auctions Require Payments

Our approach to mechanism design so far:
- Ask people what they want
- Find the best allocation we can
- Charge payments to make the incentives work out

Sometimes, we’d like to solve an allocation problem, but we don’t want to charge people money.

Examples?
Gift Exchange Example

Think of a “secret santa” or “white elephant” gift exchange.

• Everybody ends up with some item.
• People may not like the item they got.

Goal: re-allocate the items in a way that improves welfare.

Problem: we don’t want to charge people money.
Top Trading Cycle Algorithm

\[ V \leftarrow \forall \text{ all agents } \exists \]

while \( V \neq \emptyset \)

ask everyone in \( V \) whose item (in \( V \)) they want

\[ E \leftarrow \exists (i, j) \mid i \text{ wants } j's \text{ item } \exists \]

\[ C \leftarrow \text{compute-cycles}(V, E) \]

for each cycle in \( C \):

for each edge \((i, j)\) in cycle:

allocate \( j's \) item to \( i \)

\[ V \leftarrow V \setminus \exists i \exists \]

Don't let them change their mind!
Lemma

Let \( N_k \) be the set of agents removed in round \( k \).

\[
\forall i \in N_k, \ i \text{ receives their favorite item outside of } N_1 \cup N_2 \cup \ldots \cup N_{k-1}. \text{ according to } i \text{'s report}
\]

The original owner of \( i \)'s item is also in \( N_k \).

Proof?

\begin{itemize}
  \item i got what they asked for
  \item \((i, j) \in \text{cycle, so j got allocated too}\)
\end{itemize}
Theorem

The TTC algorithm is DSIC. \( i \in N_k \)

① round \( K \) \( \checkmark \) by lemma

② round \( < K \)

\[ N_1 U N_2 U \cdots U N_{K-1} \]