Consider the following congestion game:
- 20 players
- 6 facilities with corresponding cost functions
- each player must select two facilities

How big is the payoff matrix?
\[
\binom{6}{2} = 15 \quad 15^{20} \approx 3.3 \times 10^{23}
\]

How big is the payoff dict?
\[
\binom{34}{20} \approx 1.4 \times 10^9
\]
What values are still duplicated in the payoff dictionary?
Hot dogs & lemonade

What values are duplicated?
- Roles: hot dog, lemonade
Action-graph games:
- A set of players $N$
- A set of actions $A_i \forall i \in N$ (may overlap)
- An action graph $G = (V, E)$
  
  $V = \bigcup_{i \in N} A_i$ 
  edges are directed & include self-edges.

- A utility function mapping configurations to payoffs

$$(A, B, C) \quad (5, 3, 2)$$
Defn: a configuration for action $a$ is a vector of player counts for all nodes in the in-neighborhood of $a$. 

Action graph for hot dogs and lemonade:
Action sets:

How can we represent the 20-player, $(\frac{6}{2})$-strategy congestion game with an action graph?

Neighborhood size: 6

$$\sum_{i=1}^{20-p} \binom{i+9-1}{i-1} \left( \begin{array}{c} p + s - 1 \nonumber \\ p \end{array} \right)$$
deviation-payoffs \left( \text{mixed-profile, player strategy} \right)

\[
\sum_{\bar{z}} \Pr(\bar{z} | \bar{o}) \cdot u_s(\bar{z})
\]