You will work with a randomly assigned group to complete this assignment in lab on Monday. The individually-completed write-up is due by midnight on Thursday (September 20th).

(TLAGT exercise 3.4) Consider the following extension of the sponsored search setting. Each bidder $i$ now has a publicly known quality $\beta_i$, in addition to a private valuation $v_i$ per click. As usual, each slot $j$ has a CTR $\alpha_j$, and $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$. We assume that if bidder $i$ is placed in slot $j$, then the probability of a click is $\beta_i \alpha_j$. Thus bidder $i$ derives value $v_i \beta_i \alpha_j$ from the $j$th slot.

**Question 1:** Describe the welfare-maximizing allocation rule in this generalized sponsored search setting. Prove that this rule is monotone. Give an explicit formula for the per-click payment of each bidder that extends this allocation rule to a DSIC mechanism.
Question 2: Multi-Unit Auctions  In this question, we will suppose that a seller wants to auction $k$ identical copies of an item to $n > k$ bidders. First, consider the case of unit demand, where each bidder can receive at most one item, and bidder $i$ has value $v_i$ for receiving an item.

**Part A:** Describe an auction for $k$ identical items with unit demand, and prove that it is ideal.
Next consider a case with *capped demand*, where bidder $i$ has demand for up to $c_i$ items, and gets value $v_i$ for each item they receive.

**Part B:** Describe an auction for $k$ identical items with *capped demand*, and prove that it is *ideal*. 

Finally, consider a case with quantity demand, where bidder $i$ has wants a specific quantity $q_i$ of items, and gets value $v_i$ if they receive $q_i$ units.

**Part C***: Describe a DSIC auction for $k$ identical items with quantity demand, and either prove that it is ideal, or prove that no ideal auction can exist for this setting.

* This question is a bonus. You should try it, but you can get full credit without completing the proof.

**Finally:** With whom did you collaborate on this lab assignment? How much time did you spend on it (outside of lab)?