Simultaneous-Move Games

4/23/18
Recall: Game Trees

- Agents make decisions sequentially.
- Outcomes (leaves) have a utility for each agent.
- Solve by backward induction.

Good for modeling:
- Classic board games (connect four, hex, etc.)
- Discrete time interactions (resource sharing, deterrence)
Simultaneous Moves

What if agents make decisions at the same time?
• Rock-paper-scissors

```
\[
\begin{array}{ccc}
  & R & P & S \\
 R & 0,0 & -1,1 & 1,-1 \\
 P & 1,-1 & 0,0 & -1,1 \\
 S & -1,1 & 1,-1 & 0,0 \\
\end{array}
\]
```

• normal form game
• payoff matrix
Navigation Example (sequential)
Exercise: Construct a game tree.

Available actions:

- Up, Right
- Up, Left

Timing:

- short path: 30s 45s
- long path: 40s 60s

conflict: adds 20 seconds to both
Navigation Example (simultaneous)
Exercise: Fill in the payoff matrix.

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<thead>
<tr>
<th></th>
<th>U</th>
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Convention: Row player’s utility first.

- short: 30s 45s
- long: 40s 60s
- conflict: +20s

Bonus question: what if a conflict only costs 5s?
Nash Equilibrium

- An outcome where no agent can gain by unilateral deviation.

\[
\begin{array}{c|cc}
 & U & L \\
\hline
U & -60, -80 & -40, -45 \\
R & -30, -60 & -50, -65 \\
\end{array}
\]

\[
\begin{array}{c|cc}
 & U & L \\
\hline
U & -45, -65 & -40, -45 \\
R & -30, -60 & -35, -50 \\
\end{array}
\]
Identifying Nash Equilibria

for each cell in the payoff matrix:
for each player:
for each deviation:
if deviation > strategy:
cell is not a NE
if no beneficial deviations:
cell is a NE

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<td>3,3</td>
<td>6,2</td>
</tr>
<tr>
<td>C</td>
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Dominated Strategies

If strategy $X$ is always better than strategy $Y$, then $Y$ can be eliminated from the game.

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Exercise:
Iteratively eliminate dominated strategies until no more dominated strategies remain.

After one strategy is eliminated, others may be dominated in the game that remains.
A dominated strategy can never be part of a Nash equilibrium.

Why not?

If, after iterated elimination of dominated strategies, only one outcome remains, it must be a Nash equilibrium.

Why?

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