Simultaneous-Move Games

4/21/17
Recall: Game Trees

- Agents make decisions sequentially.
- Outcomes (leaves) have a utility for each agent.
- Solve by backward induction.

Good for modeling:
- Classic board games (connect four, hex, etc.)
- Discrete time interactions (resource sharing, deterrence)
Simultaneous Moves

What if agents make decisions at the same time?

- Rock-paper-scissors

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>R</td>
</tr>
<tr>
<td>1</td>
<td>0,0</td>
</tr>
<tr>
<td>2</td>
<td>1,-1</td>
</tr>
<tr>
<td>S</td>
<td>-1,1</td>
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- normal form game
- payoff matrix
Navigation Example (sequential)
Exercise: Construct a game tree.

Available actions:
- Up, Right
- Up, Left

Timing:
- short path: 30s, 45s
- long path: 40s, 60s

Conflict: adds 20 seconds to both
Navigation Example (simultaneous)
Exercise: Fill in the payoff matrix.

Convention:
Row player’s utility first.

Bonus question: what if a conflict only costs 5s?
Nash Equilibrium

• An outcome where no agent can gain by unilateral deviation.

\[\begin{array}{c|c|c}
\hline
& U & L \\
\hline
U & -60, -80 & -40, -45 \\
R & -30, -60 & -50, -65 \\
\hline
\end{array}\]
Identifying Nash Equilibria

for each cell in the payoff matrix:
   for each player:
      for each deviation:
         if deviation > strategy:
            cell is not a NE
      if no beneficial deviations:
         cell is a NE

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<tbody>
<tr>
<td>A</td>
<td>9,3</td>
<td>1,4</td>
<td>7,3</td>
</tr>
<tr>
<td>B</td>
<td>4,1</td>
<td>3,3</td>
<td>6,2</td>
</tr>
<tr>
<td>C</td>
<td>-1,9</td>
<td>2,8</td>
<td>8,-1</td>
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Dominated Strategies

If strategy $X$ is always better than strategy $Y$, then $Y$ can be eliminated from the game.

After one strategy is eliminated, others may be dominated in the game that remains.

Exercise: Iteratively eliminate dominated strategies until no more dominated strategies remain.
Pursuit/Evasion Example
Exercise: Construct a payoff matrix.

Available actions:

- Right
- Left

Incentives:

- +5 (catch)
- +3
- 0
- -1 (miss)
Exercise: Identify all Nash equilibria.

- There are no pure-strategy equilibria!
- We need mixed strategies
  - Agents should deliberately randomize their actions

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<tr>
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<td>0,1</td>
<td>5,-1</td>
</tr>
<tr>
<td>R</td>
<td>3,-1</td>
<td>0,1</td>
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More than two agents

• Add more dimensions to the array.
• Add more utilities to each outcome tuple.

In a game with \( N \) agents that each have \( S \) actions, how many utilities does the payoff matrix contain?