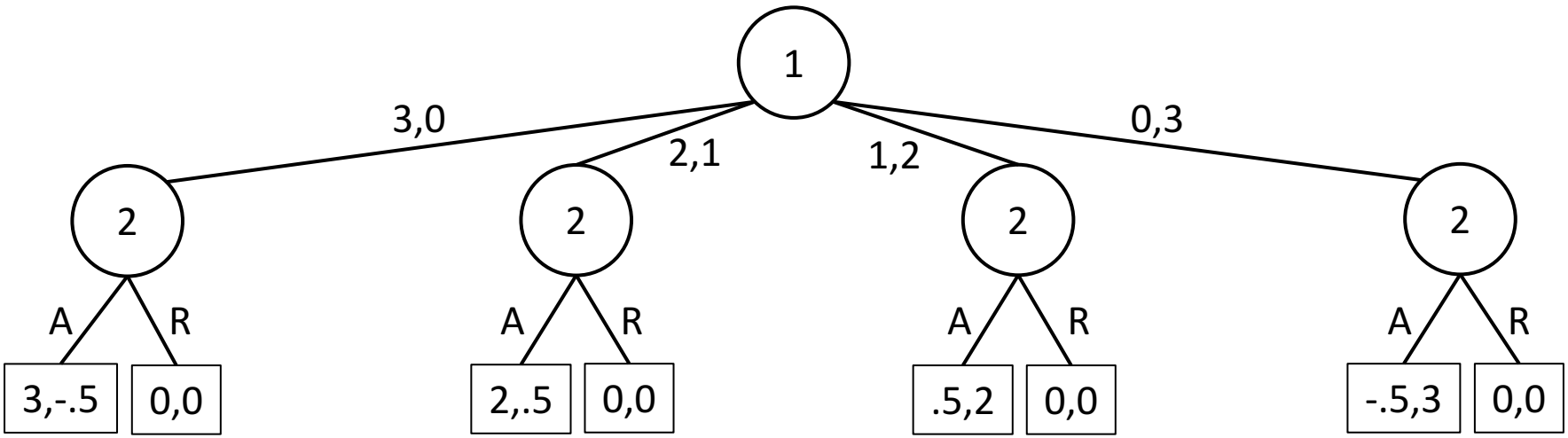
# Simultaneous-Move Games

4/21/17

# Recall: Game Trees




- Agents make decisions sequentially.
- Outcomes (leaves) have a utility for each agent.
- Solve by backward induction.

Good for modeling:

- Classic board games (connect four, hex, etc.)
- Discrete time interactions (resource sharing, deterrence)

# Simultaneous Moves

What if agents make decisions at the same time?

- Rock-paper-scissors

| | | 2 | | |
|---|---|---|---|---|
| | | **R** | **P** | **S** |
| 1 | **R** | 0,0 | -1,1 | 1,-1 |
| | **P** | 1,-1 | 0,0 | -1,1 |
| | **S** | -1,1 | 1,-1 | 0,0 |

- normal form game
- payoff matrix

# Navigation Example (sequential)

# Exercise: Construct a game tree.

Available actions:

 Up, Right

 Up, Left

Timing:

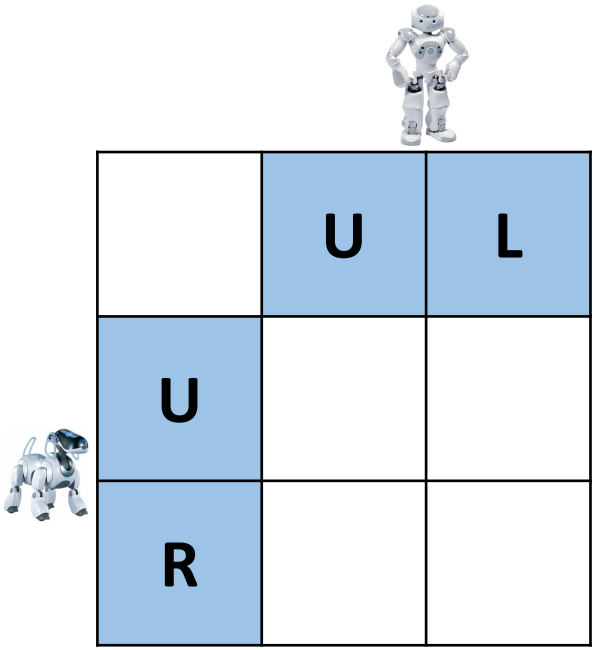
| |  |  |
|---|---|---|
| short path: | 30s | 45s |
| long path: | 40s | 60s |

conflict: adds 20 seconds to both

# Navigation Example (simultaneous)

# Exercise: Fill in the payoff matrix.

Convention:
Row player's utility first.



| | U | L |
|---|---|---|
| U | | |
| R | | |

Bonus question: what if a conflict only costs 5s?

# Nash Equilibrium

- An outcome where no agent can gain by unilateral deviation.

| | U | L |
|---|---|---|
| U | -60,-80 | -40,-45 |
| R | -30,-60 | -50,-65 |

| | U | L |
|---|---|---|
| U | -45,-65 | -40,-45 |
| R | -30,-60 | -35,-50 |

# Identifying Nash Equilibria

```
for each cell in the payoff matrix:
    for each player:
        for each deviation:
            if deviation > strategy:
                cell is not a NE
    if no beneficial deviations:
        cell is a NE
```

| | D | E | F |
|---|---|---|---|
| A | 9,3 | 1,4 | 7,3 |
| B | 4,1 | 3,3 | 6,2 |
| C | -1,9 | 2,8 | 8,-1 |

# Dominated Strategies

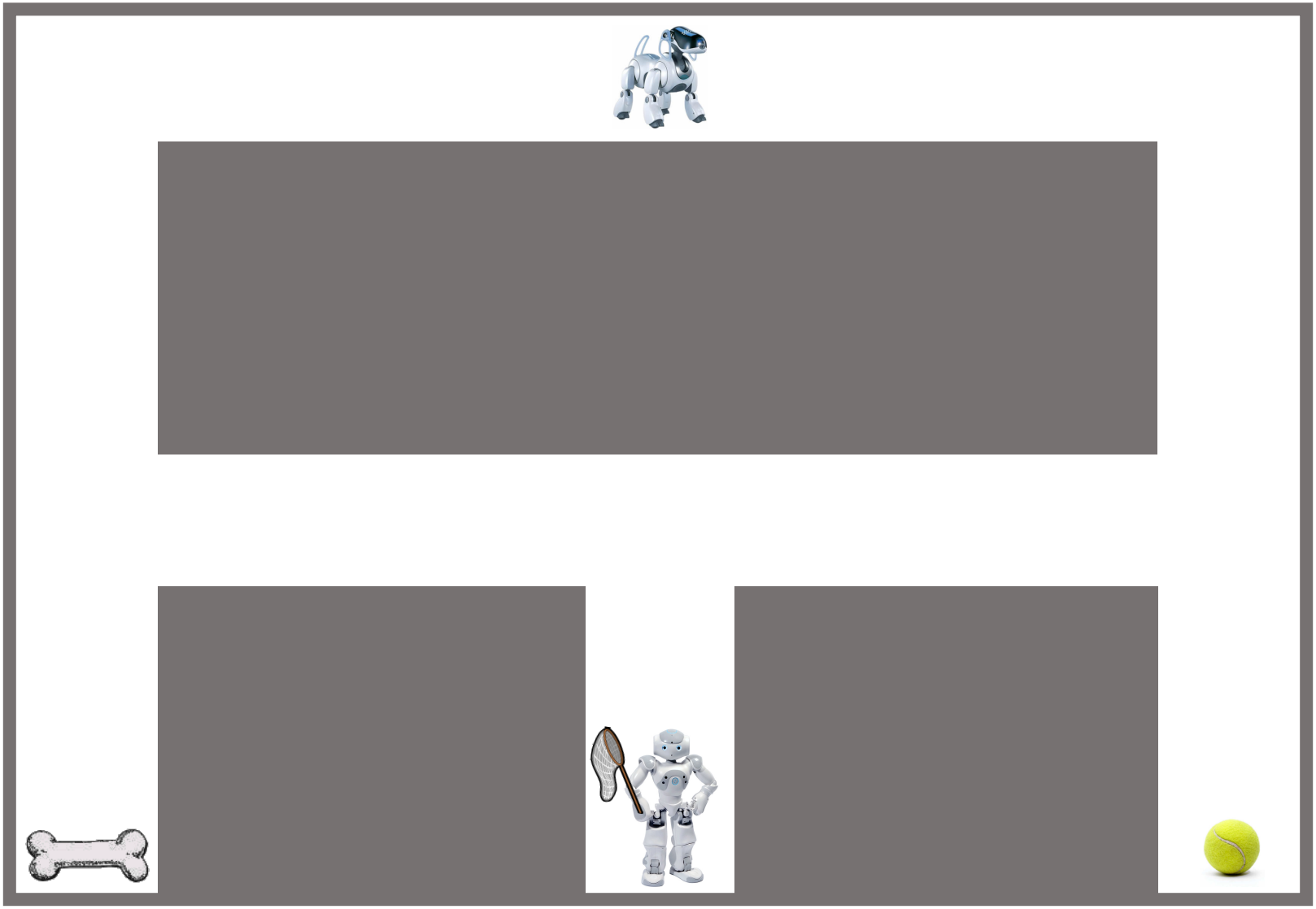
If strategy *X* is always better than strategy *Y*, then *Y* can be eliminated from the game.

| | D | E | F |
|---|---|---|---|
| A | 9,3 | 1,4 | 7,5 |
| B | 4,1 | 3,3 | 6,2 |
| C | -1,9 | 2,8 | 8,-1 |

Exercise:
Iteratively eliminate dominated strategies until no more dominated strategies remain.

After one strategy is eliminated, others may be dominated in the game that remains.

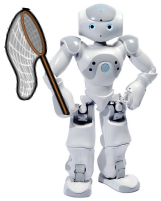
# Pursuit/Evasion Example



# Exercise: Construct a payoff matrix.

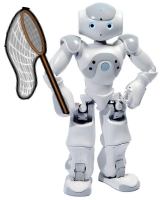
Available actions:

Left, Right



Left, Right

Incentives:



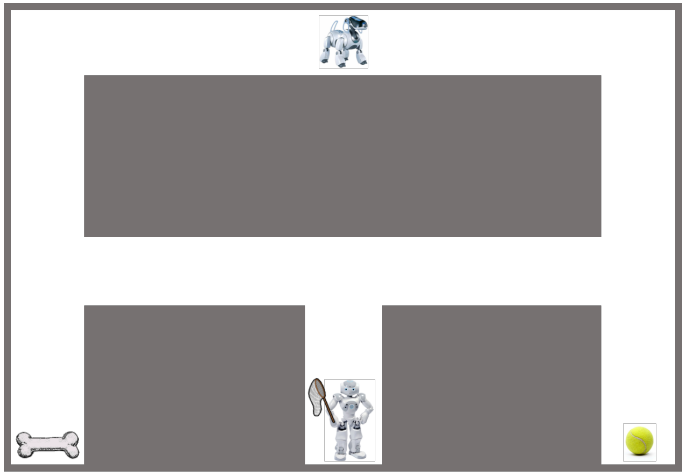
+5

+3

0

catch: +1

miss: −1



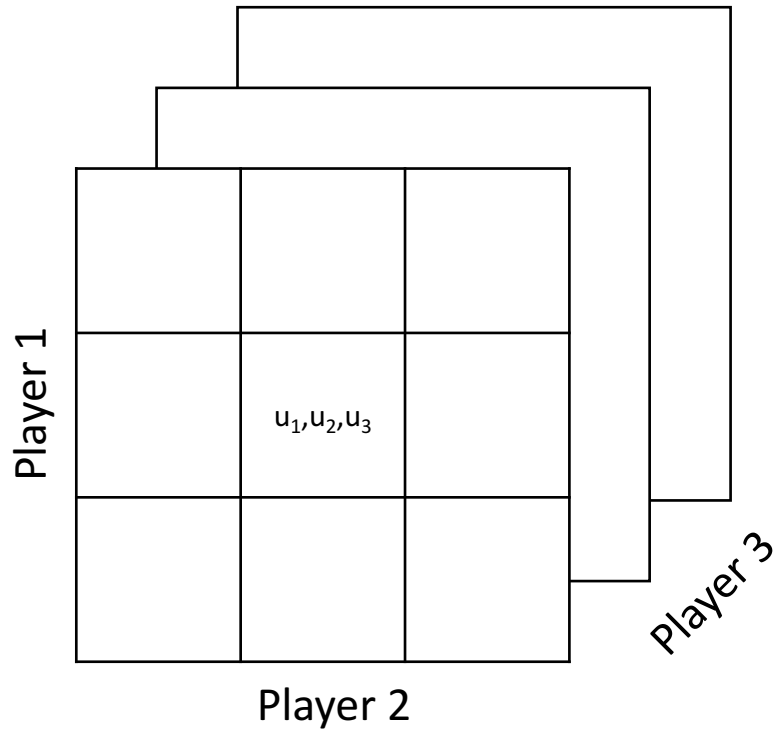
# Exercise: Identify all Nash equilibria.

| | L | R |
|---|---|---|
| **L** | 0,1 | 5,-1 |
| **R** | 3,-1 | 0,1 |

- There are no pure-strategy equilibria!
- We need mixed strategies
  - Agents should deliberately randomize their actions

# More than two agents

- Add more dimensions to the array.
- Add more utilities to each outcome tuple.




In a game with *N* agents that each have *S* actions, how many utilities does the payoff matrix contain?