Local Search

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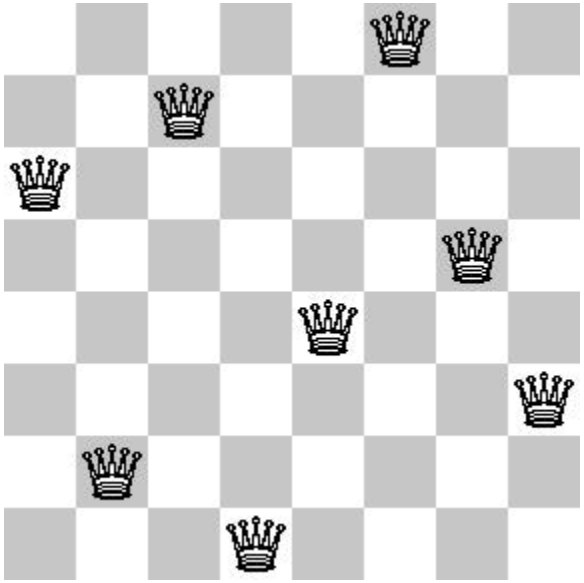
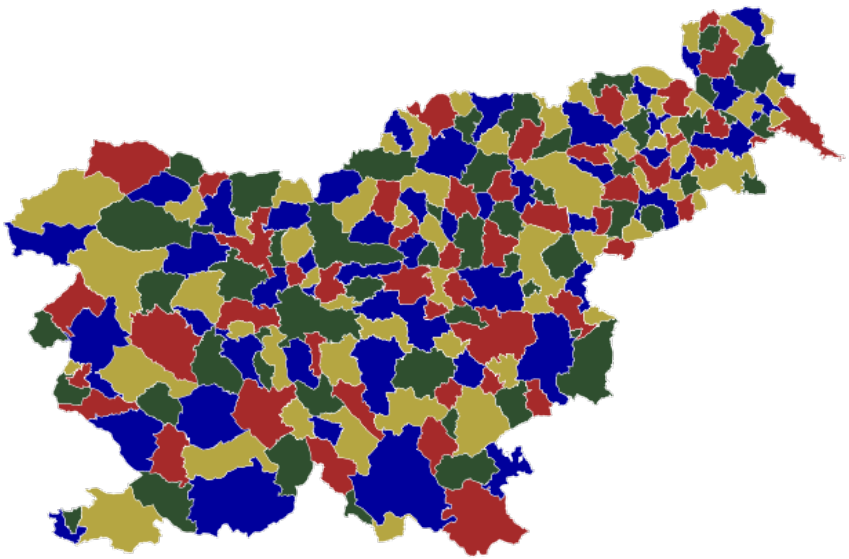
Consider the following problems

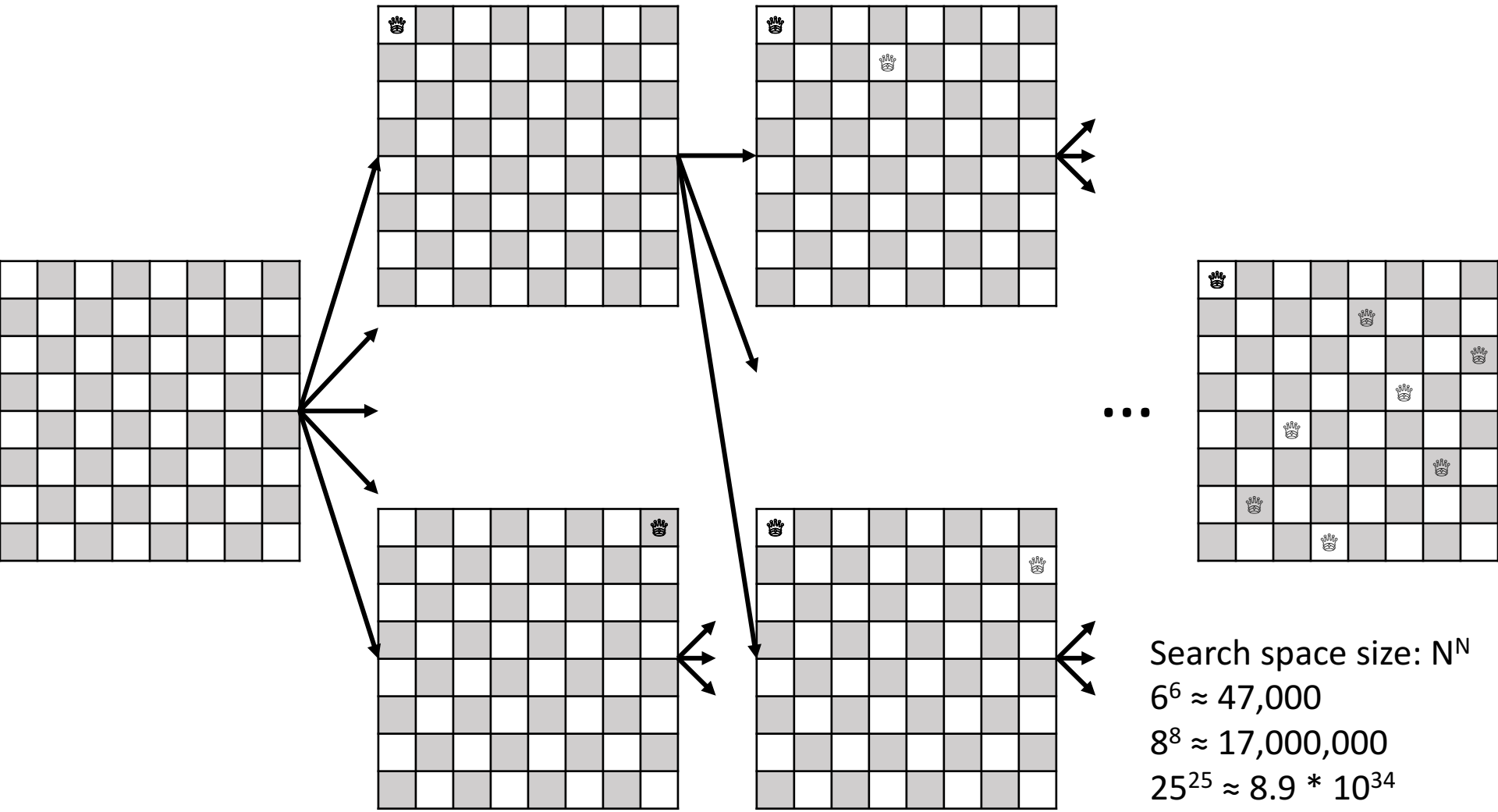




<u>N-Queens</u>: place n queens on an n x n chessboard so that none share a row, column, or diagonal.

<u>K-Coloring</u>: find an assignment of k colors to graph nodes (map regions) so that no adjacent nodes share a color.

N-Queens as state space search



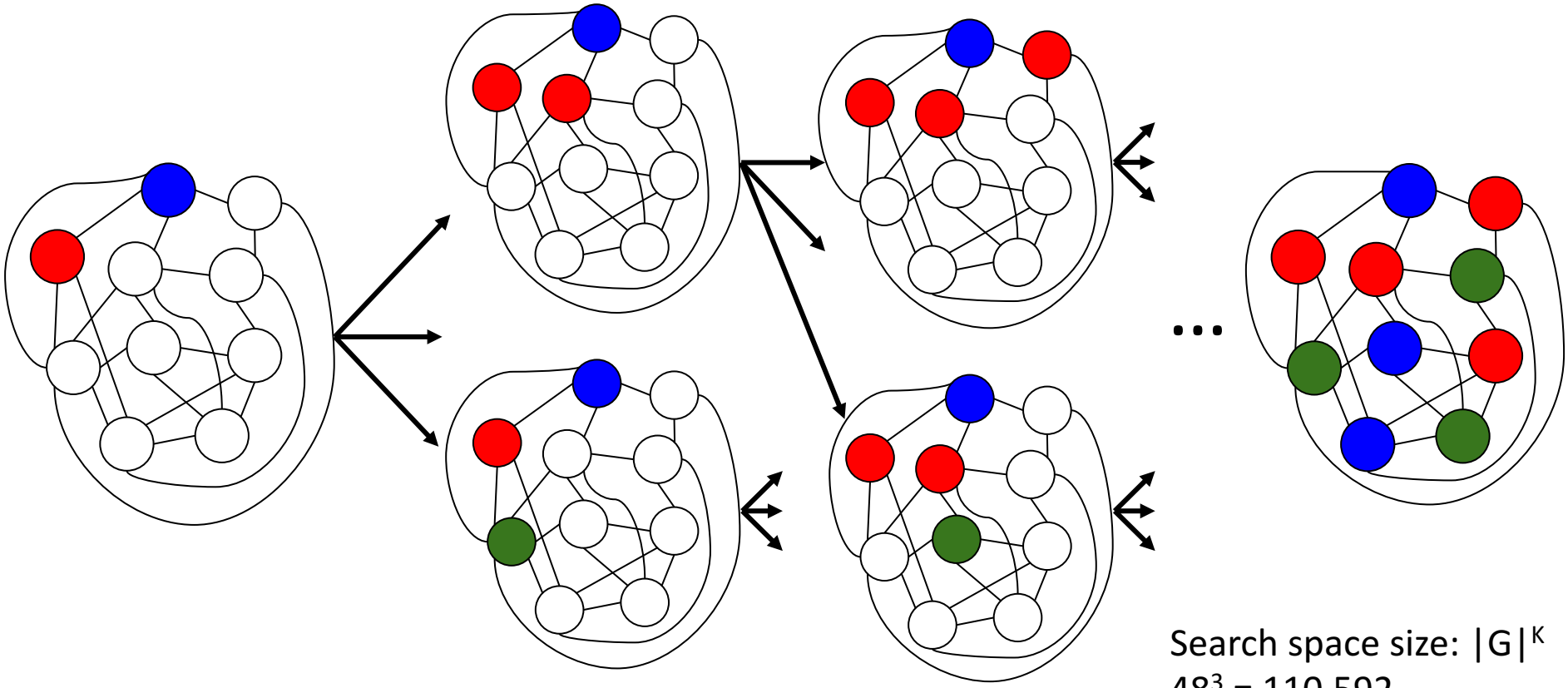
Search space size: N^N

$6^6 \approx 47{,}000$

$8^8 \approx 17{,}000{,}000$

$25^{25} \approx 8.9 * 10^{34}$

K-Coloring as state space search



Search space size: $|G|^K$

$48^3 = 110{,}592$

$10{,}000^8 = 10^{32}$

A different approach

State Space Search

- Search for paths to goals.
- Given a start state, goal state, operators.
- Apply operators to states to generate new states.

examples: BFS, DFS, UCS, A*, iterative deepening

Local Search

- Search for solutions.
- Given candidate solutions.
- Make small adjustments to candidates to generate new, potentially better candidates.

examples: hill climbing, simulated annealing, beam search, genetic algorithms

Requirements for local search

- We don’t care about the *path* to a solution.
- We can generate and evaluate candidate solutions.
- We can generate neighbor candidates by small modifications.
- Similar candidates have similar values.

N-Queens Local Search Representation

Candidate solution: an assignment of 1 queen per row.

can represent as a tuple: (0,1,2,5,6,7,2,3)

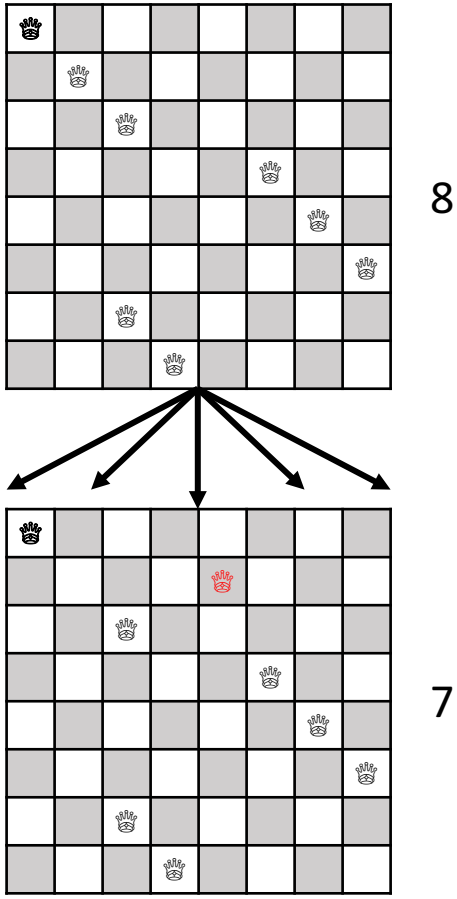
Neighbors: boards differing by the placement of 1 ♕.

each state has N*(N-1) neighbors

Evaluation: minimize number of attacked queens.

v(0,1,2,5,6,7,2,3) = 8

v(0,4,2,5,6,7,2,3) = 7



Hill Climbing

Key idea:

- Start with an arbitrary candidate.
- Iteratively move to a neighbor with higher value.
  - Climb up the value hill.

cost = -value

Basic pseudocode:

```
state = random_candidate()
neighbor = None
while cost(state) > cost(neighbor):
  state = neighbor
  neighbor = best_neighbor(state)
```

Trace of successful run

```
state cost: 8 neighbor cost: 6
state cost: 6 neighbor cost: 5
state cost: 5 neighbor cost: 4
state cost: 4 neighbor cost: 3
state cost: 3 neighbor cost: 2
state cost: 2 neighbor cost: 0
state cost: 0 neighbor cost: 1
-----------------
| |Q| | | | | | |
| | | | |Q| | | |
| | | | | | |Q| |
| | | |Q| | | | |
|Q| | | | | | | |
| | | | | | | |Q|
| | | | | |Q| | |
| | |Q| | | | | |
-----------------
Goal found in 7 steps
```

Trace of unsuccessful run

```
state cost: 6 neighbor cost: 5
state cost: 5 neighbor cost: 4
state cost: 4 neighbor cost: 3
state cost: 3 neighbor cost: 2
state cost: 2 neighbor cost: 2
-----------------
| |Q| | | | | | |
| | | | |Q| | | |
| | | | | | | |Q|
| | | |Q| | | | |
| | | | | | |Q| |
| | | |Q| | | | |
| | | | | |Q| | |
|Q| | | | | | | |
-----------------
Goal NOT found in 5 steps
```

Getting stuck in local optima

random restarts would help

random steps would help

Improved hill climbing pseudocode

```
state = random_candidate()
best_state = state
best_cost = cost(state)
for i=1:MAX_ITERS:
  if rand() < RESTART_PROB:
    state = random_candidate()
  else if rand() < RAND_STEP_PROB:
    state = random_neighbor(state)
  else:
    state = best_neighbor(state)
  if cost(state) < best_cost:
    best_cost = cost(state)
    best_state = state
```

Trace of improved pseudocode

```
cost: 7 best: 7
cost: 6 best: 6
cost: 5 best: 5
cost: 4 best: 4
cost: 2 best: 2
cost: 2 best: 2
cost: 2 best: 2
cost: 2 best: 2
cost: 2 best: 2
cost: 2 best: 2
Hill climbing restarted (1)...
cost: 7 best: 2
cost: 5 best: 2
cost: 4 best: 2
cost: 2 best: 2
cost: 2 best: 2
cost: 2 best: 2
cost: 0 best: 0
```

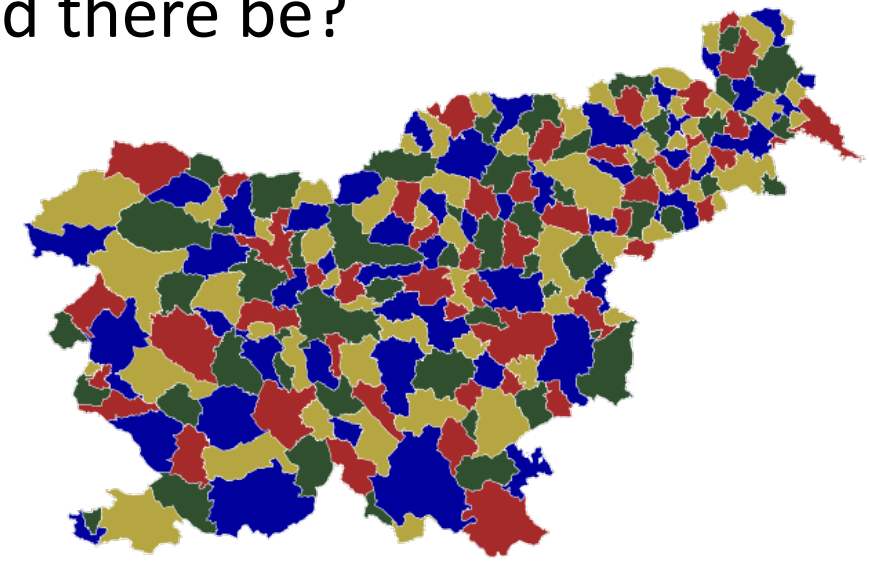
```
-----------------
| | | | | |Q| | |
| | |Q| | | | | |
| | | | | | |Q| |
| | | |Q| | | | |
|Q| | | | | | | |
| | | | | | | |Q|
| |Q| | | | | | |
| | | | |Q| | | |
-----------------
Goal found in 18 steps
```

This version can solve 25-queens

```
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |
|   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|
|   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |
|   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |
|   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |
|   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |
|   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |
|   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |
|Q|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |   |   |   |   |Q|   |   |   |   |   |   |   |   |   |   |   |   |
```

Exercise: map coloring

- How would you represent states?
- How big is the state space?
- What is the objective function?
- What is the successor function?
- How many successors would there be?



Simulated Annealing

- Always select moves randomly.
- Accept if improving.
- Accept bad moves with some probability.

Key idea: gradually reduce the probability of accepting bad moves.

parameters:

- initial temperature
- decay rate

$$\text{prob} = e^{\Delta / T}$$

$$\Delta = \text{cost(state)} - \text{cost(neighbor)}$$

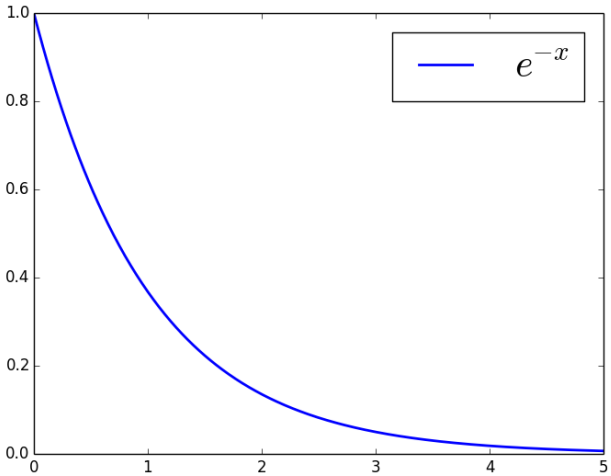
$$T = \text{initial\_temp} * (\text{decay\_rate})^{\text{rounds}}$$

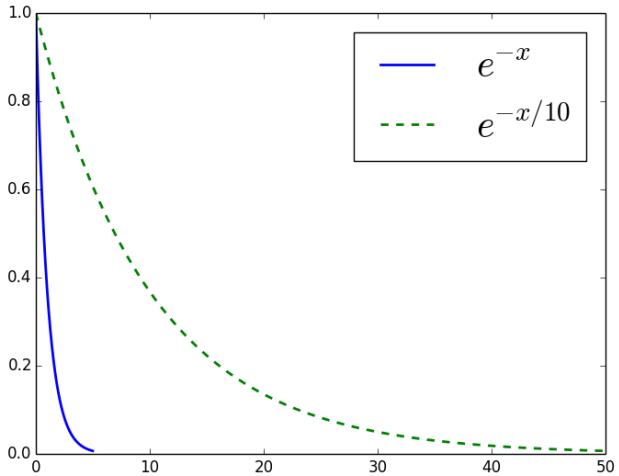
Simulated annealing pseudocode

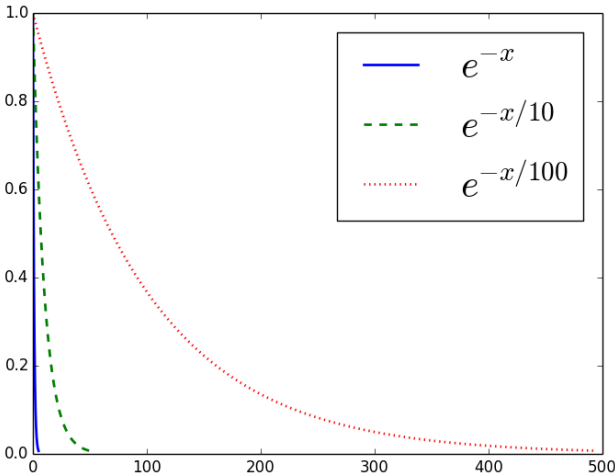
```
state = random_candidate()
best_state = state
temp = INIT_TEMP
for round = 1:MAX_ITERS:
  neighbor = random_step(state)
  if cost(neighbor) < cost(best_state)
    best_state = neighbor
  if accept(state, neighbor, temp)
    state = neighbor
  temp *= DECAY
```

```
function accept(state, neighbor, temp):
   delta = cost(state) - cost(neighbor)
   r ~ U[0,1]
   return r < e^(delta / temp)
```

Effect of initial temperature







Effect of decay rate