Example

for $i = 1 \ldots n$ **do**
  for $j = 1 \ldots i$ **do**
    do work
  end for
end for

How many times do we "do work"?

$$1 + 2 + 3 + \ldots + n = \sum_{i=1}^{n} i$$

Claim: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. Proof (by induction):
Base Case: $n = 1$.

$$\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$$

Inductive Case: show that if $\sum_{i=1}^{k} = \frac{k(k+1)}{2}$ then $\sum_{i=1}^{k+1} = \frac{(k+1)(k+2)}{2}$.

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$
$$1 + 2 + \ldots + n = \frac{k(k+1)}{2}$$
$$1 + 2 + \ldots + n + (n+1) = \frac{k(k+1)}{2} + (k+1)$$
$$\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} + (k+1)$$
$$\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$
$$\sum_{i=1}^{k+1} i = \frac{k(k+1) + 2(k+1)}{2}$$
$$\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$$

So by induction, our formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ holds for all positive integers n.

Exercise

for $i = 1 \ldots n$ **do**
  for $j = 1 \ldots i^2$ **do**
    do work
  end for
end for

How many times do we "do work"?

$$1 + 4 + 9 + \ldots + n^2 = \sum_{i=1}^{n} i^2$$

Show using induction that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1}{6}$.

Example

```
function Fact(n)
    if n == 0 then
        return 1
    end if
    return n * Fact(n − 1)
end function
```

Claim: this function correctly computes factorial. Proof (by induction):
Base case: $n = 0$.

$$\textsc{Fact}(0) = 1 = 0!$$

Inductive case:

- Assume: $\textsc{Fact}(k) = k!$
- Goal: $\textsc{Fact}(k+1) = (k+1)!$

$$\begin{aligned}
\textsc{Fact}(k+1) &= \textsc{Fact}(k) * (k+1) \\
\textsc{Fact}(k+1) &= k! * (k+1) \\
\textsc{Fact}(k+1) &= (k+1)!
\end{aligned}$$

So by induction, our function Fact computes factorial correctly for any non-negative integer n.

Exercise

```
function contains(arr, n, val)
    if n == 0 then
        return false
    end if
    if arr[n-1] == val then
        return true
    end if
    return contains(arr, n-1, val)
end function
```

Show using induction that contains correctly determines whether the array *arr* contains the value *val* for any array, length (n), and value.