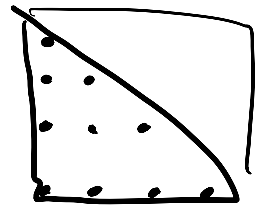
```
for i = 1 ··· n :
    for j = 1 ··· i :
        do work
```



$\sim n^2$

$$1 + 2 + 3 + \dots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$\forall n \geq 1$

Proof by induction $\quad P(n)$

- Base case $\quad P(1)$
- Inductive case $\quad P(k) \Rightarrow P(k+1)$
  - ↳ assume $P(k)$, show $P(k+1)$ follows

Base case $n=1$

$$\sum_{i=1}^{1} i = 1 \qquad \frac{1(1+1)}{2} = 1 \checkmark$$

Inductive case

assume $\displaystyle\sum_{i=1}^{K} i = \frac{K(K+1)}{2}$

$$\underline{(K+1) + 1 + 2 + 3 + \cdots + K} = \frac{K(K+1)}{2} + (K+1)$$

$$\sum_{i=1}^{K+1} i = \frac{K(K+1) + 2(K+1)}{2}$$

$$= \frac{K^2 + K + 2K + 2}{2}$$

$$= \frac{K^2 + 3K + 2}{2}$$

$$= \frac{(K+1)(K+2)}{2}$$

use this to show

$$\sum_{i=1}^{K+1} i = \frac{(K+1)(K+2)}{2}$$

for $i=1 \cdots n$:

  for $j = 1 \cdots i^2$:

    do work

$$1 + 4 + 9 + 16 \cdots + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Base case:

$n=1$

$$\frac{1(1+1)(2+1)}{6} = \frac{1 \cdot 2 \cdot 3}{6} = 1 = \sum_{i=1}^{1} i^2$$

Inductive Case:

if $P(k)$ then $P(k+1)$

$$\frac{k(k+1)(2k+1)}{6} = \sum_{i=1}^{k} i^2 \leftarrow \text{inductive hypothesis}$$

$$(k+1)^2 + \frac{k(k+1)(2k+1)}{6} = \sum_{i=1}^{k+1} i^2$$

$$\frac{6(k^2+2k+1)}{6} + \frac{2k^3+3k^2+k}{6}$$

$$2k^3+9k^2+13k+6$$

$$\frac{(k+)(k+2)(2k+3)}{6} = \sum_{i=1}^{k+1} i^2$$

Prove Correctness

```
function fact(n):        k+1
    if n == 0:
        return 1
    return n * fact(n-1)
```

k

IH: $k!$

W.t.S: $\forall n \geq 0 \quad fact(n) = n!$

Base case: $n = 0$

$0! = 1$

$fact(0) = 1$ ✓

Inductive case:

Assume: $fact(k) = k!$

goal: $fact(k+1) = (k+1)!$

$$fact(k+1) = (k+1)\,k! = (k+1)!$$

```
function contains(arr, n, val):
    if n == 0:
        return False
    if arr[n-1] == val:
        return True
    return contains(arr, n-1, val)
```

Base: $n = 0$ ✓

Induction: