Tournament

n rounds

$2^n - 1$ total games

round	1	2	3	$\dots$	$\overset{k+1}{n}$
games	$1 +$	$2 +$	$4 +$		$+\underset{k}{2^{n-1}}$

$$\sum_{i=0}^{n-1} 2^i = 2^n - 1$$

Proof by induction:

$n=1$ $\quad \displaystyle\sum_{i=0}^{1-1} 2^i = 2^0 = 1$ (base case) inductive case: $n \geq 1$

$P(k) \Rightarrow P(k+1)$

assume show

Inductive hypothesis: true for k

$$\sum_{i=0}^{k-1} 2^i = 2^k - 1$$

Goal:

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

$$2^k + \sum_{i=0}^{k-1} 2^i = 2^k - 1 + 2^k$$

$$2^k + 1 + 2 + 4 + \cdots 2^{k-1} = 2 \cdot (2^k) - 1$$

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

$\forall n \in \mathbb{Z} \quad n \geq 1$

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Function power(base, exponent):
    if exponent == 0:
        return 1
    end if
    return base * power(base,
                        exponent-1)
end Function
```

base exponent

- For any base $\in \mathbb{Z}$
  any eponent $\in \mathbb{Z}$ exponent ≥ 0

b^e

Base case: exponent $= 0$

$base^0 = 1$ ✓

Inductive case:

Assume: $power(base, k) = base^k$

want: $power(base, k+1) = base^{k+1}$

$$\underbrace{\text{base} \cdot \text{power}(\text{base}, k)}_{\text{base}^k} = \text{base}^k \cdot \text{base}$$
$$= \text{base}^{k+1}$$

$x = 1$
for $i = 1 \dots$ exponent
    x *= base
end for
return x

$x = \text{base}^i$

Invariant:
- object
- loop
- recursive

Find (value) // returns true if value in list

false otherwise