The Probabilistic Method

Week 9: Random Graphs

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CS49/Math59
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Reading Quiz

In a bin are **n** balls, numbered **1, 2, 3, ..., n-2, n-1, n**. **r** of the balls are colored **blue**. How many ways are there to pick **t** balls from the bin without replacement?

(A) **(n choose t)**

(B) **(n choose r)**

(C) **$\sum_{k=0}^{r}$ (r choose k)*(n-r choose t-k)**

(D) **$\sum_{k=0}^{r}$ (r choose k)*(n-r choose k)**

(E) **multiple answers correct**

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Zero-One Laws

Definition: **fix $0 < p < 1$.** Property P obeys 0-1 law if
$\lim_{n\to\infty} \Pr[G(n,p) \text{ has } P] = 0 \text{ or } 1$.

Examples:

- **G** has **triangle**
- **G** has **no isolated vertex**
- **G** has **diameter < 2**

Theorem: fix **$0 < p < 1$.** **Any property** expressed in **first-order theory of graphs** obeys **0-1** law.

Clicker Question

What property does the following sentence represent?

$\exists x\ \exists y\ \exists z\ [\neg(x=y) \wedge \neg(x=z) \wedge \neg(y=z) \wedge x\sim y \wedge x\sim z \wedge y\sim z]$

(A) G has a triangle

(B) G has diameter ≤ 2

(C) G has no isolated vertex

(D) G has an independent set of size ≤ 3

(E) None of the above

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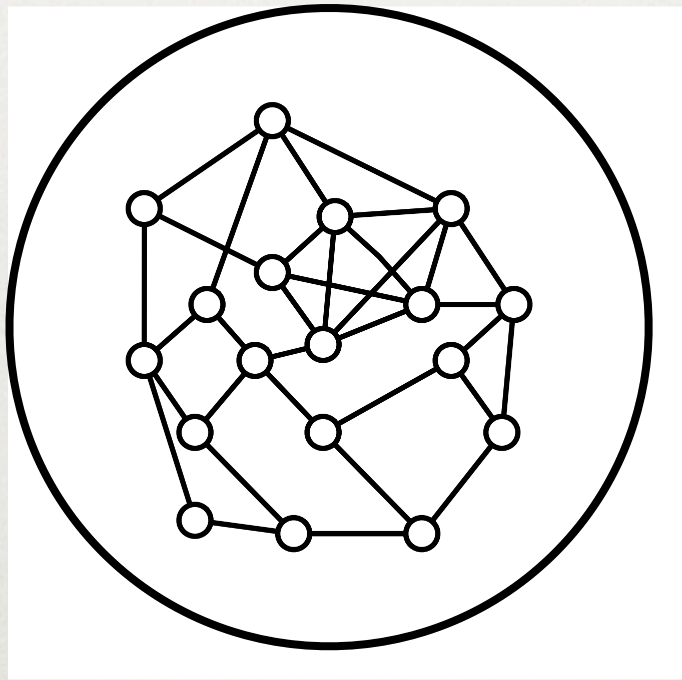
Concentration of Measure for Clique Number

Definition: **CL(G)** := size of largest clique in **G**



Theorem: **If G ~ G(n,p) then almost always**

CL(G(n,p)) ~ 2log(n)/log(1/p).

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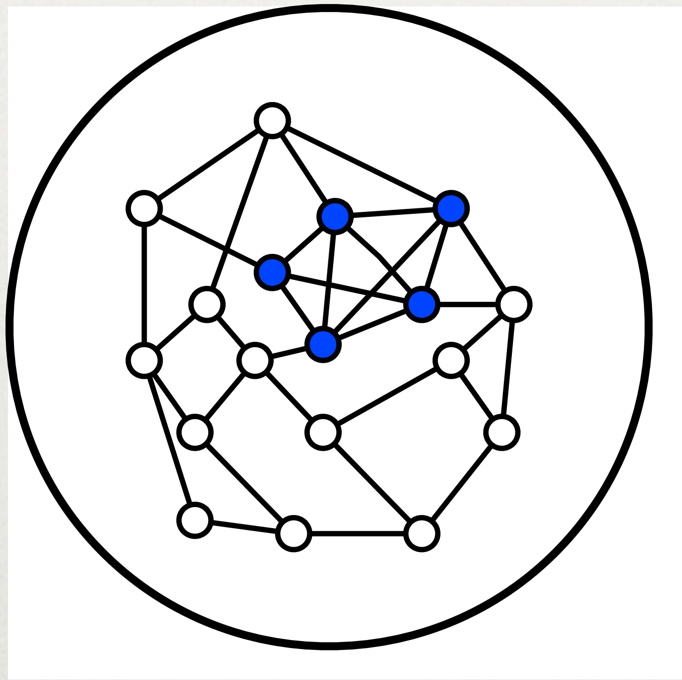

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Clicker Question

Suppose that **$r = c\log(n)$** for some constant **c**.

What is **$E[Y_r]$**?

(A) $E[Yr] \approx 2^{c(1-c/2)\log(n)}$

(B) $E[Yr] \approx (n \text{ choose } \log(n)) * 2^{-(c\log(n) \text{ choose } 2)}$

(C) $E[Yr] \approx 2^{c\log(n)\text{^}2 - (c\text{^}2/2)*\log(n)\text{^}2}$

(D) Multiple answers correct

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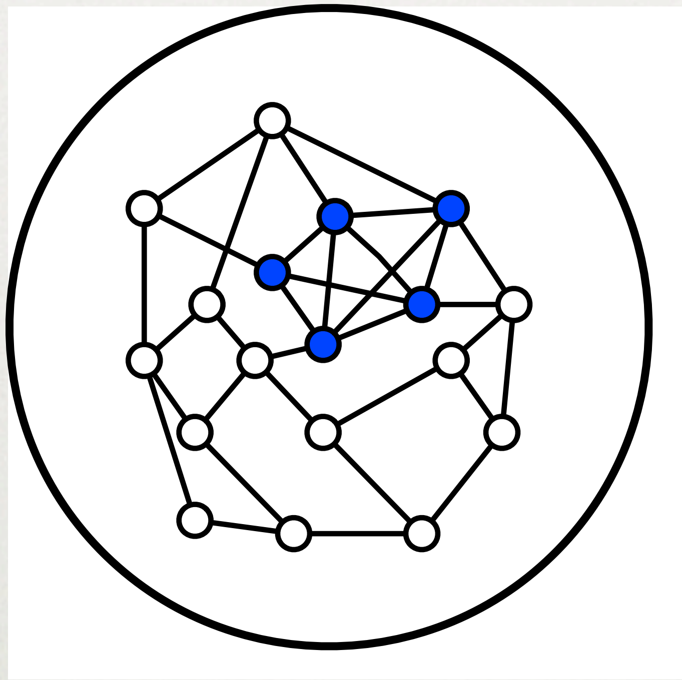
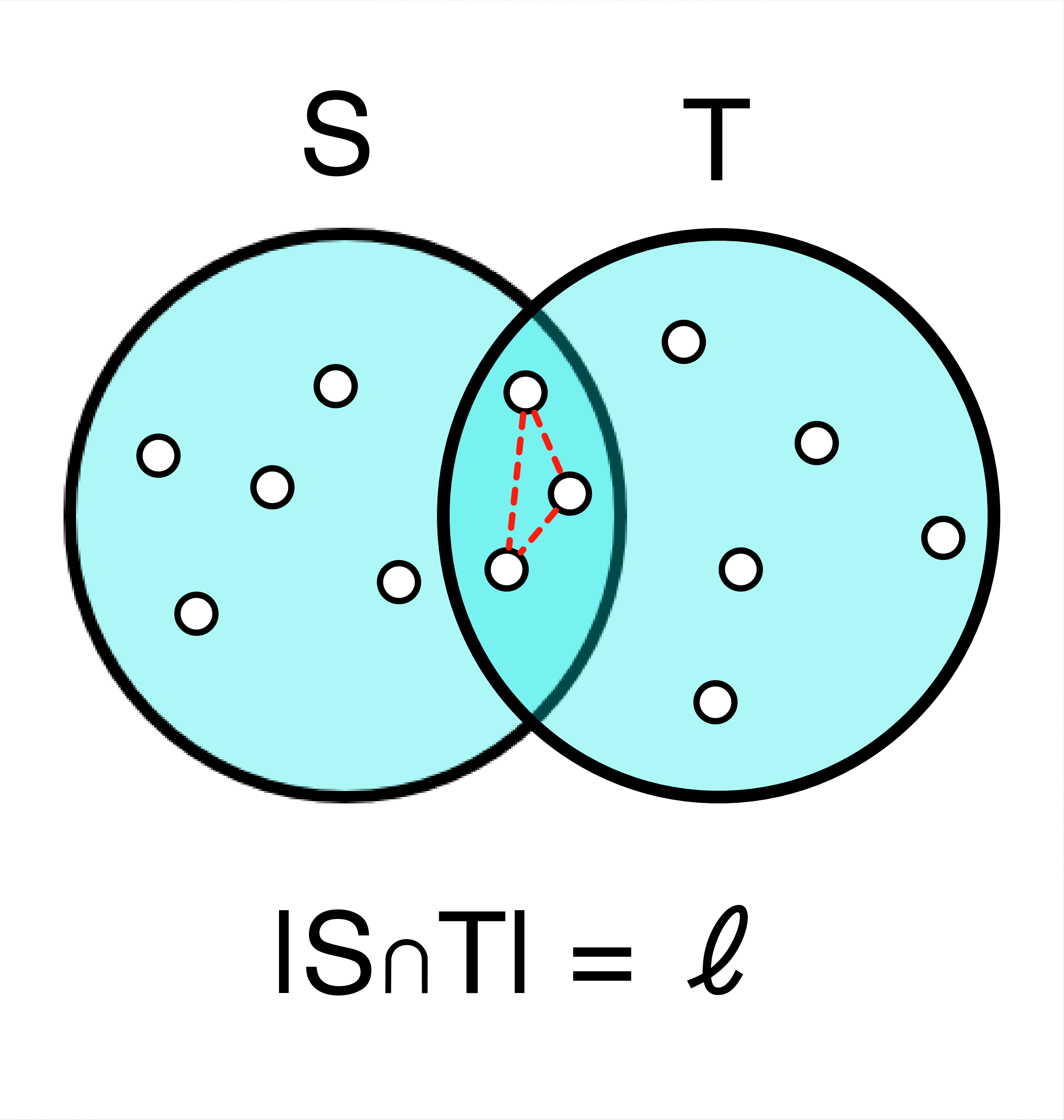


Results:

(1) **If r = 2log(n) then almost always CL(G) < r**

(2) **If r = 2(1-o(1))log(n) then a.a. CL(G) > r**

Estimating $E[Y_R^2]$



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