THE PROBABILISTIC METHOD WEEK 8: SECOND MOMENT METHOD



JOSHUA BRODY CS49/MATH59 FALL 2015

READING QUIZ

Let X be a nonnegative integer random variable. Which of the following inequalities hold?

(A) $Pr[X > 0] \leq Var[X]/E[X]$

(B) $Pr[X > 0] \le E[X]$

(C) $Pr[X=0] \leq Var[X]/E[X]^2$

(D) Multiple answers correct

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MOST INTEGERS HAVE In(In(n)) PRIME FACTORS

Theorem: Let g(n) be a function growing arbitrarily slowly. Then, there are o(n) integers $x \le n$ such that

| *ν* (x) - ln(ln(n))| > g(n) √ln(ln(n))

Proof:

- Take random $\mathbf{x} \in [\mathbf{n}]$
- Estimate # distinct primes dividing x
- Use Chebyshev to bound deviation.

CLICKER QUESTION

What is **E[X_pX_q]**?

(A) $E[X_pX_q] = 1/pq$

(B) $1/pq - 1/n \le E[X_pX_q] \le 1/pq$

(C) $1/p + 1/q - 1/n \le E[X_pX_q] \le 1/p + 1/q$

(D) $1/p + 1/q \le E[X_pX_q] \le 1/pq$

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