The Probabilistic Method

Week 7: Alterations

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CS49/Math59
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Reading Quiz

What is the danger with recoloring?

(A) **If the recoloring is too weak, not all errors removed**

(B) **If the recoloring is too strong, new errors created**

(C) **Both (A) and (B)**

(D) **None of the above**

Reading Quiz

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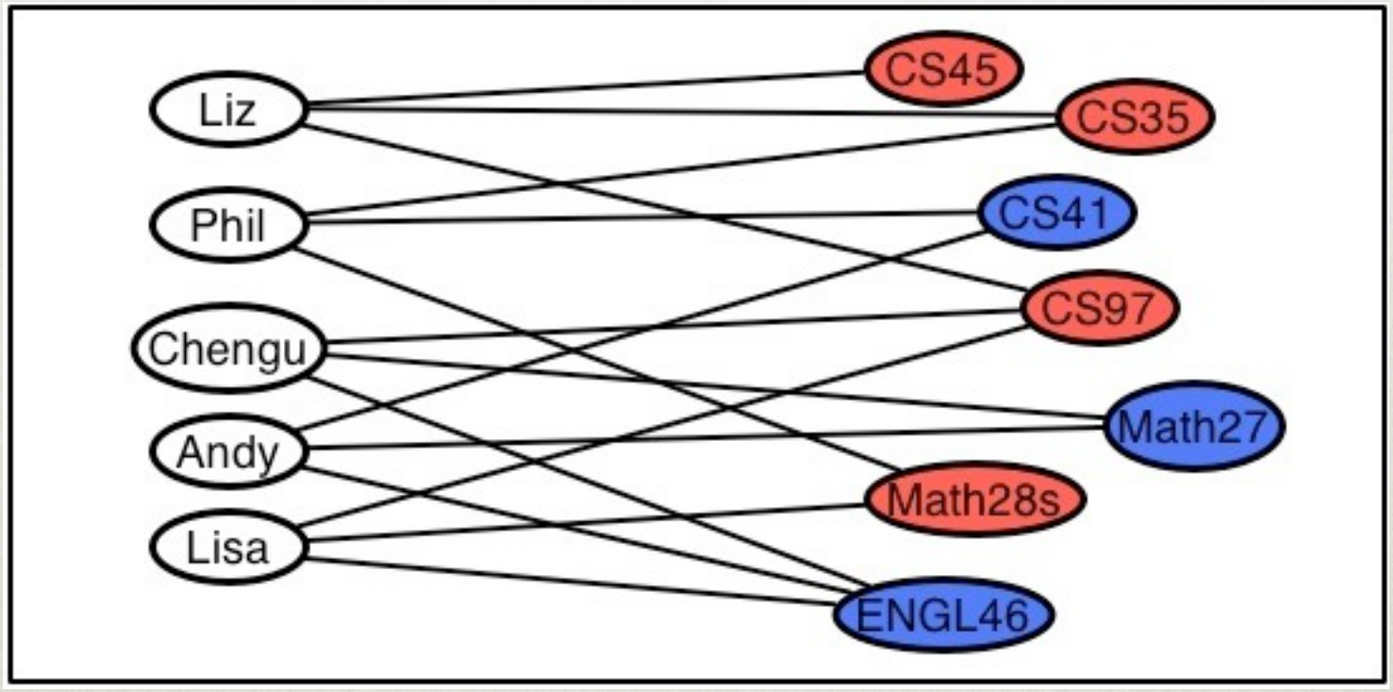
Course Registration

- **m** students, each registers for **n** classes Spring 2016.
- classes are **MWF** or **TTh**
- students don't want all **MWF** or all **TTh**

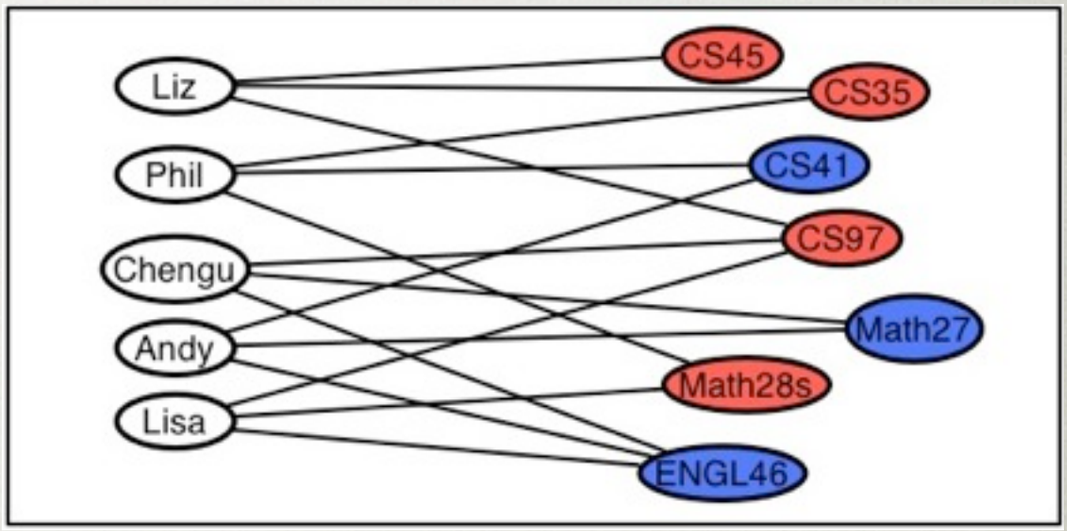


Question: Is there a way to schedule courses **MWF** or **TTh** so no student has all courses on same day?

Basic Method

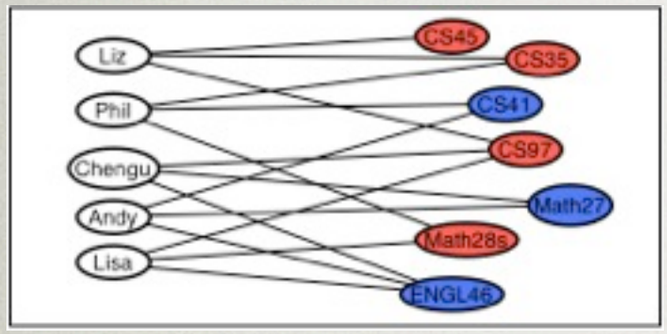
Uniformly color classes **MWF** or **TTh**
BAD$_S$: all courses of student **S** are **MWF** or all **TTh**.

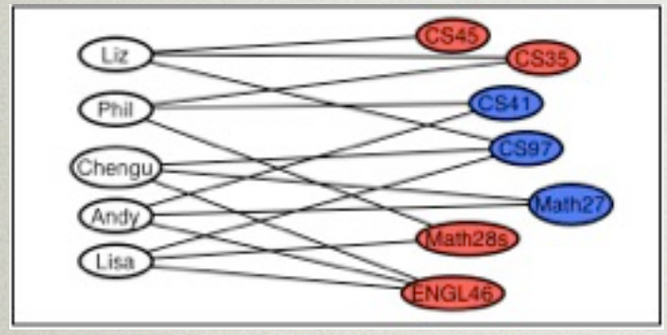
- **Pr[BAD$_S$] = $2*2^{-n}$**
- **#students = m**
- union bound:
  Pr[BAD] ≤ $m2^{-(n-1)}$
- want: **Pr[BAD] < 1**



Conclusion: If **$m2^{-(n-1)} < 1$** then there is schedule so no student has all classes on same day.

Alterations





(1) First, generate uniform coloring
- assign each class **MWF** or **TTh**

(2) Second, ***recolor***
- if class registered by m-c student, change color **w/prob p**

(3) return class schedule

Question: What can go wrong? When can we get monochromatic students?

Clicker Question

Let A_S be event that

(i) student **S** was monochromatic after first coloring

(ii) none of **S**'s courses changed during recolor.

What is $\Pr[A_S]$?

(A) $\Pr[A_S] \le 2^{-n}p^n$

(B) $\Pr[A_S] \le 2^{-n}(1-p)^n$

(C) $\Pr[A_S] \le p^n(1-p)^n$

(D) $\Pr[A_S] \le 2*2^{-n}(1-p)^n$

(E) None of the above

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