The Probabilistic Method

Week 7: Alterations

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CS49/Math59
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Reading Quiz

What is the alteration technique?

(A) **Generate GOOD object by choosing random object, then removing any badness "by hand"**

(B) **Generate GOOD object by choosing random object, then showing BAD probability < 1**

(C) **Generate GOOD object by choosing random object, then using Chernoff Bound**

(D) **Choose edges of graph by alternating between red edges and blue edges**

(E) **None of the above**

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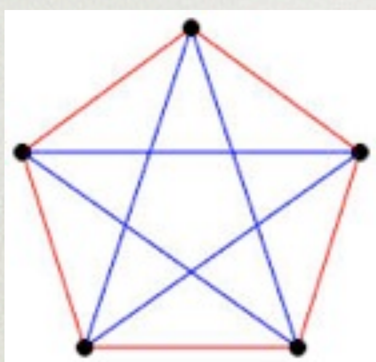
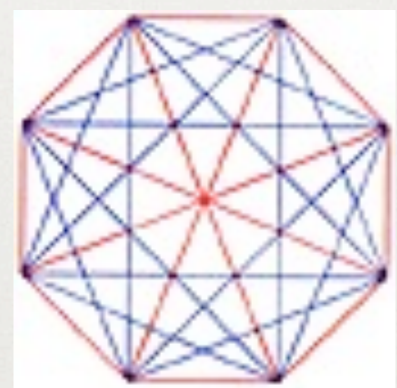
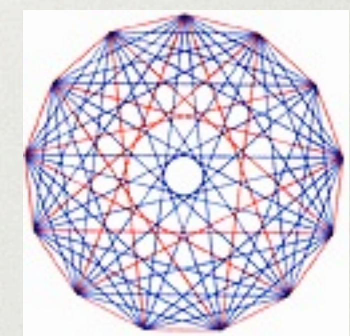
Ramsey Theory

R(k,k) := smallest **n** such that for every two-coloring of K_n, there is red K_k subgraph or a blue K_k subgraph.







Basic Method: If $\binom{n}{k} 2^{1-\binom{k}{2}} < 1$ then $R(k,k) > n$.

Alterations: $R(k,k) > n - \binom{n}{k} 2^{1-\binom{k}{2}}$

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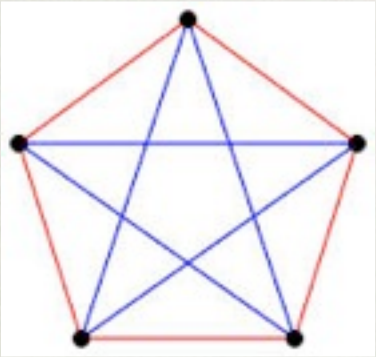
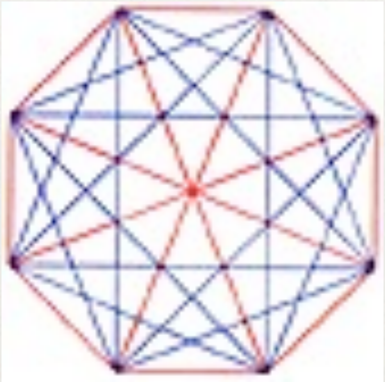
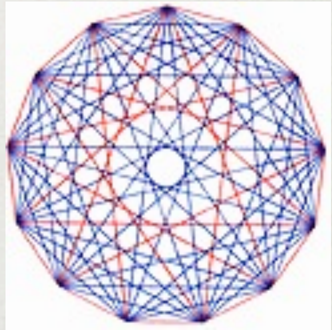






Basic Method: $R(k,k) > (1+o(1))\, k2^k/e\sqrt{2}$

Alterations: $R(k,k) > (1+o(1))\, k2^k/e$

Clicker Question

Will alterations always give improvement over basic probabilistic method approach?

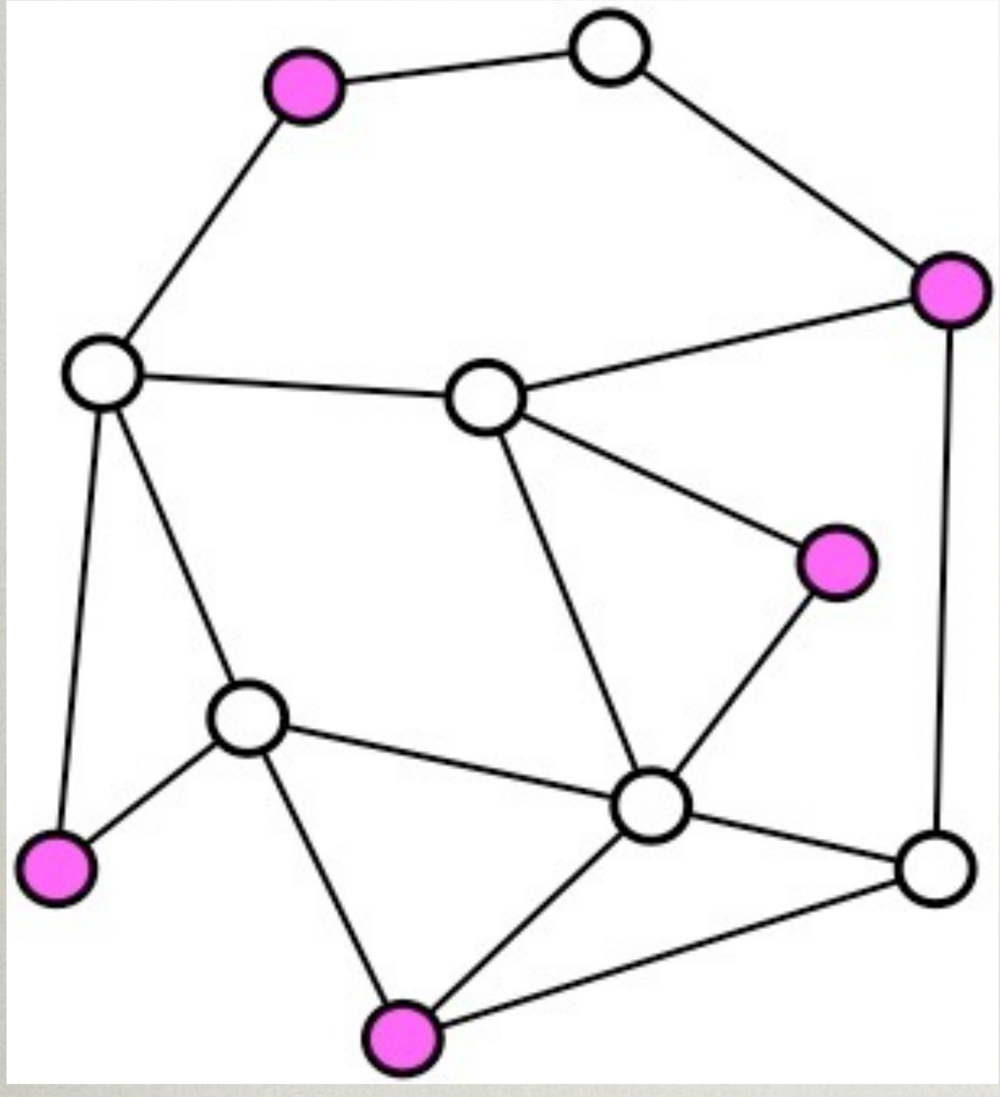
(A) **Yes**

(B) **No**

(C) **Maybe**

(D) **None of the above**

Independent Sets



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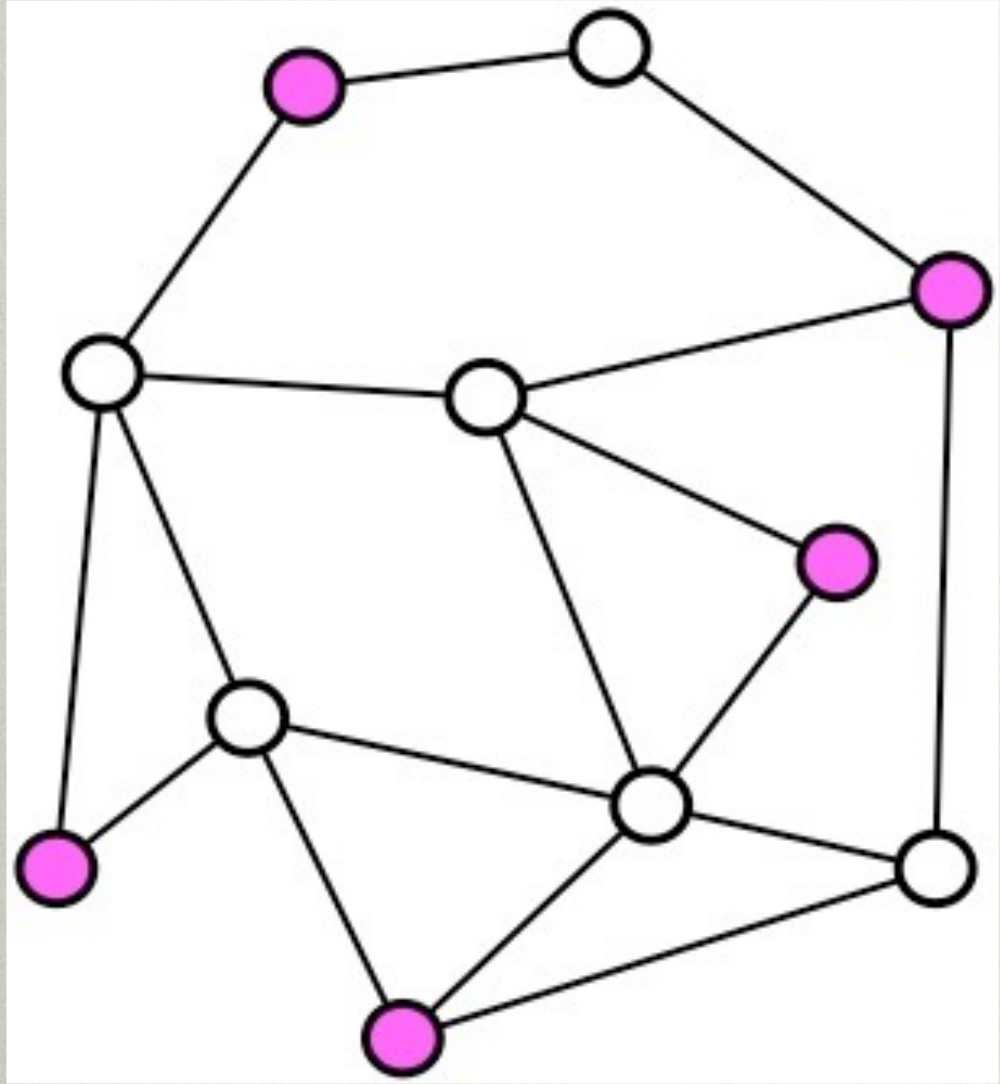
- **$\alpha(G)$:** size of largest independent set

Independent Sets



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Theorem: If **$G = (V,E)$** is a graph with **n** vertices and **$nd/2$** edges, then **$\alpha(G) \geq n/2d$**

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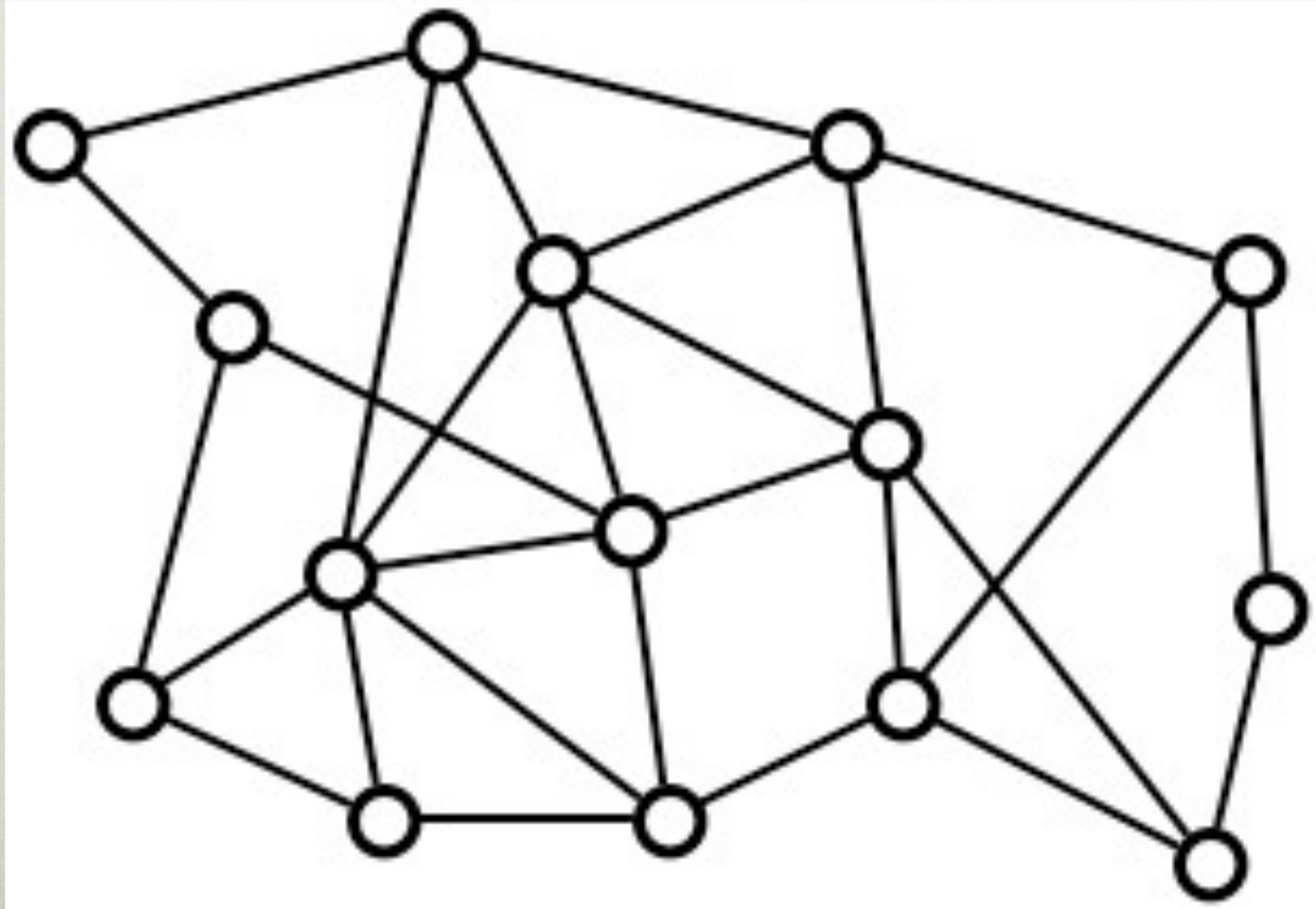

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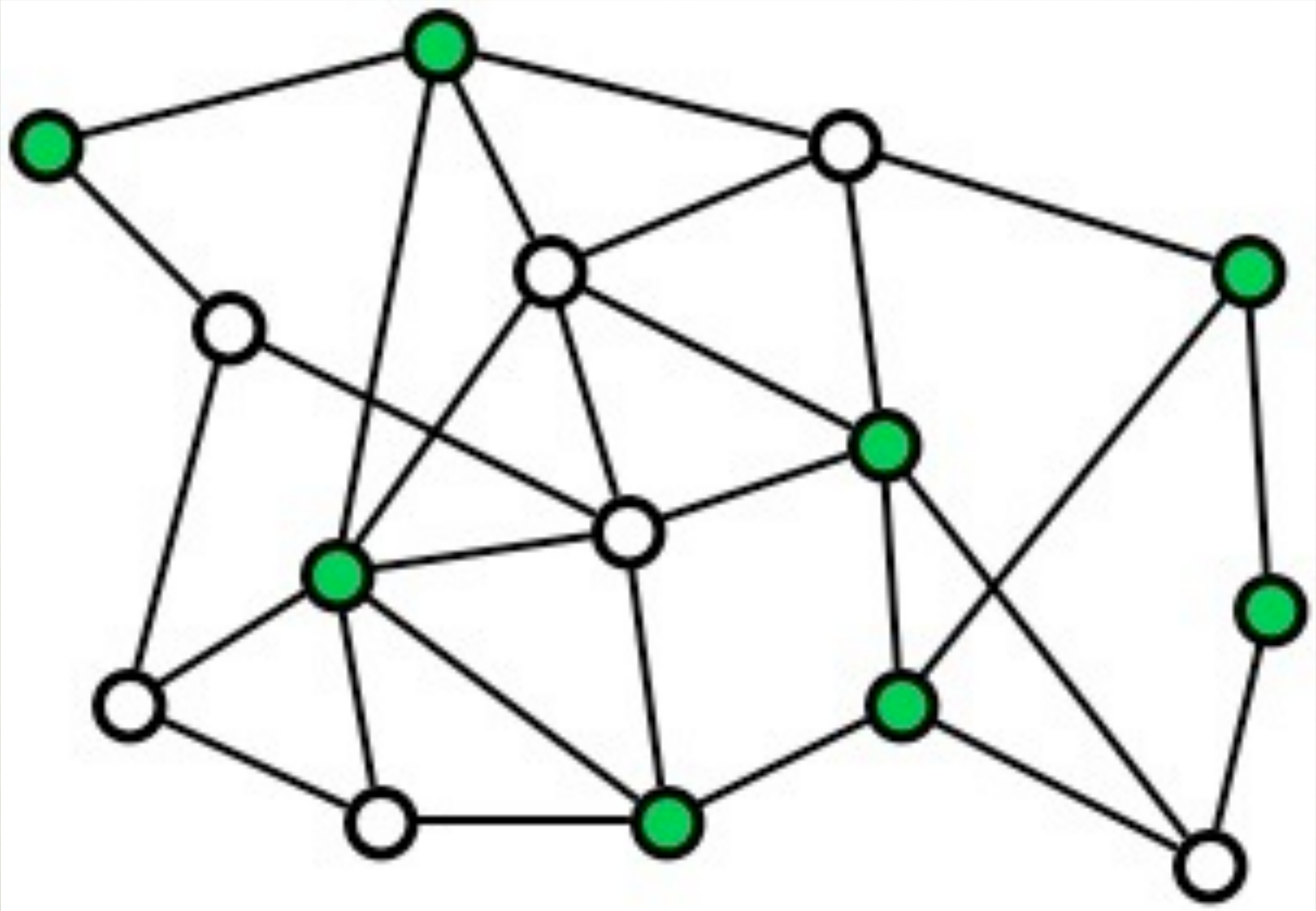

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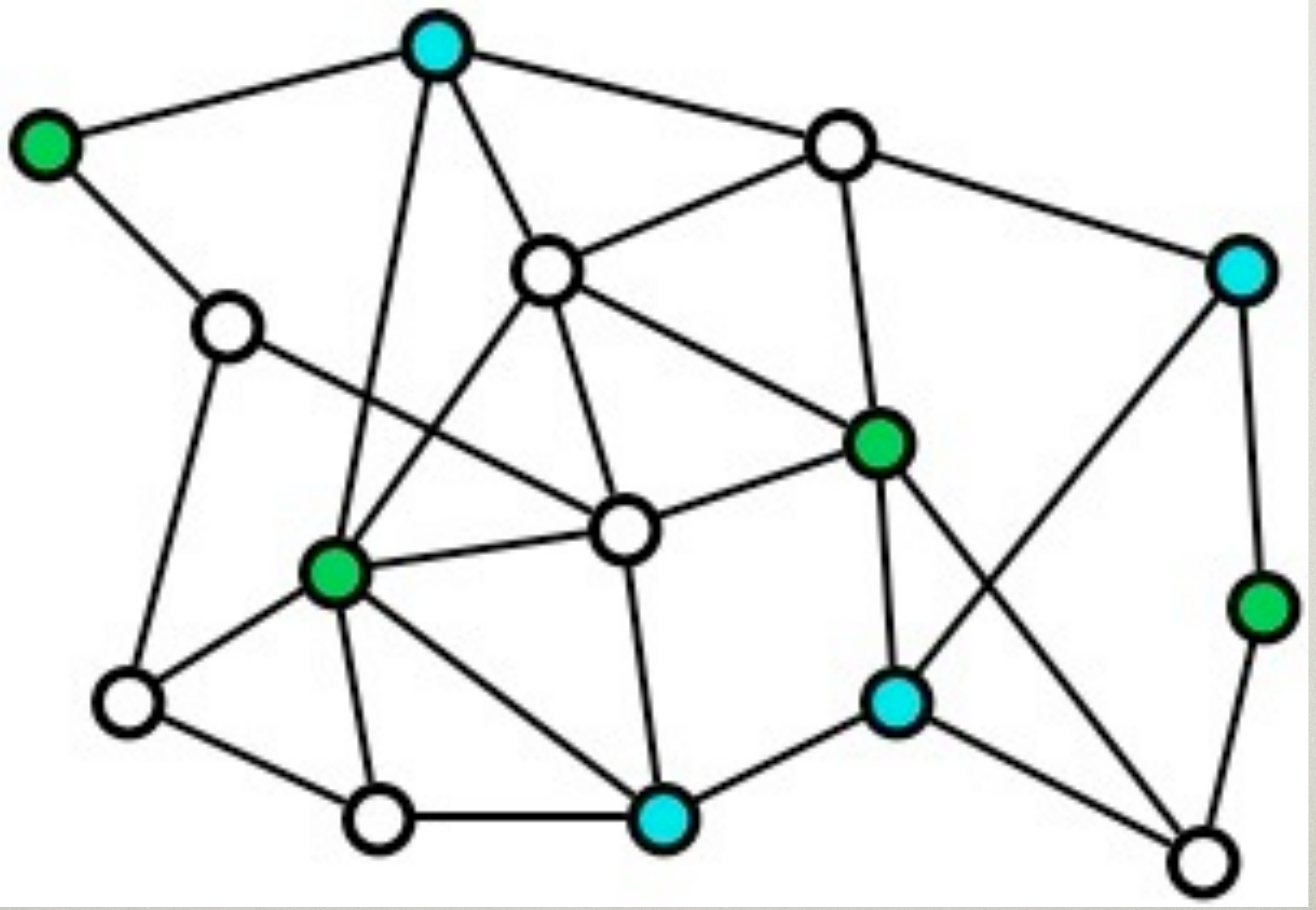

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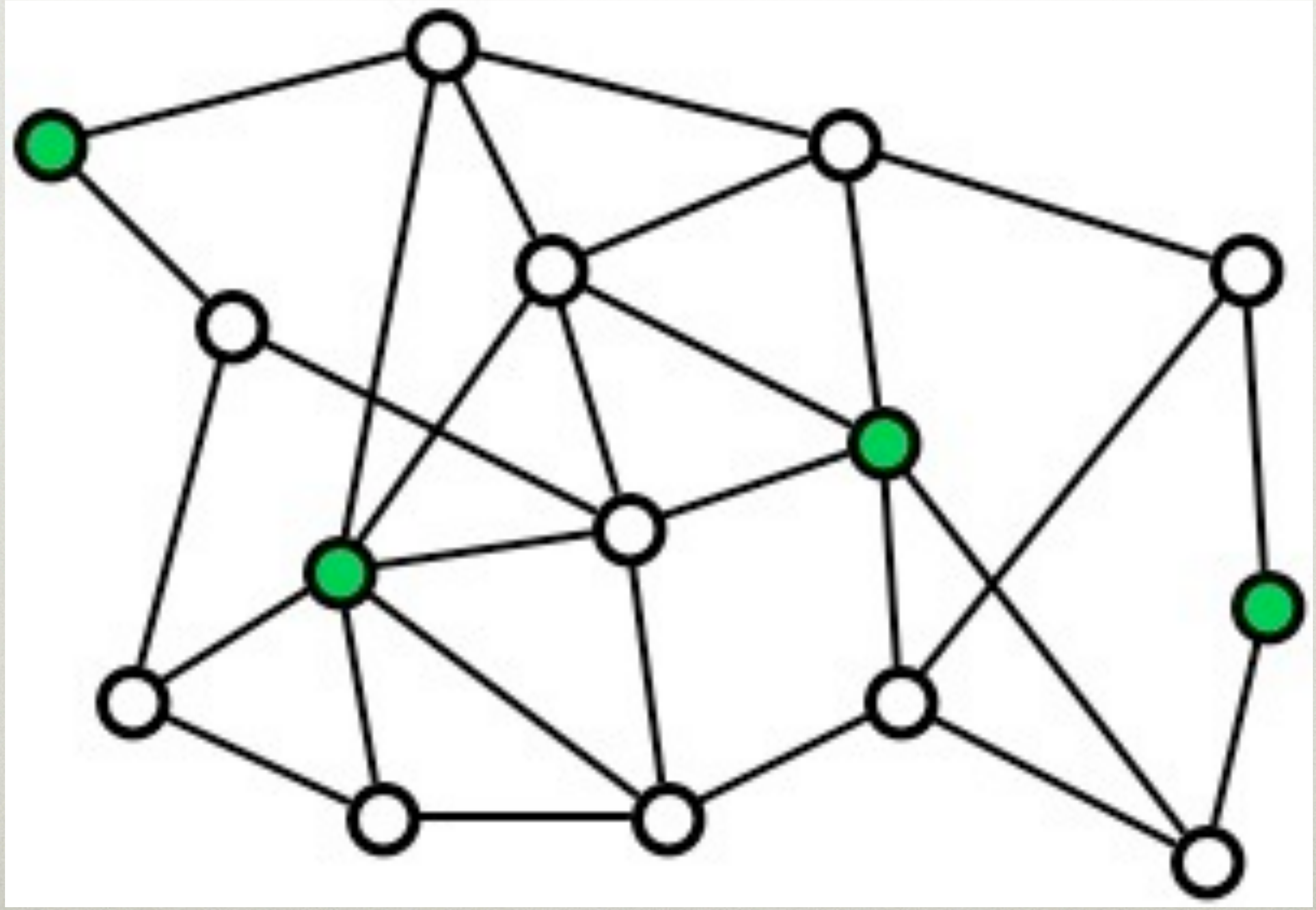

Clicker Question

What is **E[Y]**?

(A) **E[Y] = (nd/2)*p**

(B) **E[Y] = nd*p**

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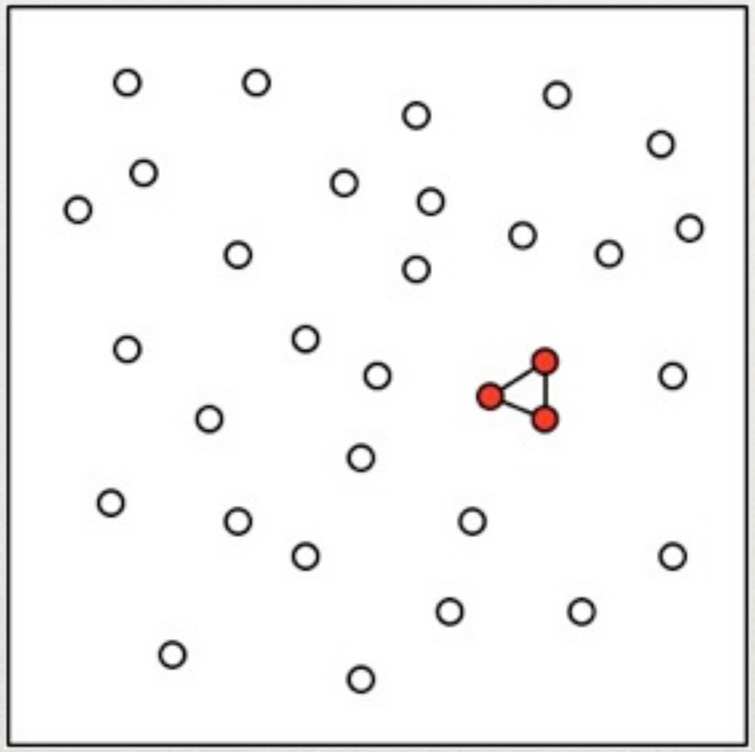
Minimizing Area

Let **S** be a set of **n** points on unit square **[0,1]x[0,1]**

$$T(S) := \min_{P,Q,R \in S} \text{area}(PQR)$$



$$T(n) := \max_S T(S)$$

Theorem: $T(n) = \Omega(1/n^2)$

Clicker Question

What is **E[#triangles w/area ≤ $1/100n^2$]**?

(A) **E[#small triangles] ≤ (n choose 3)*16*π/$100n^2$**

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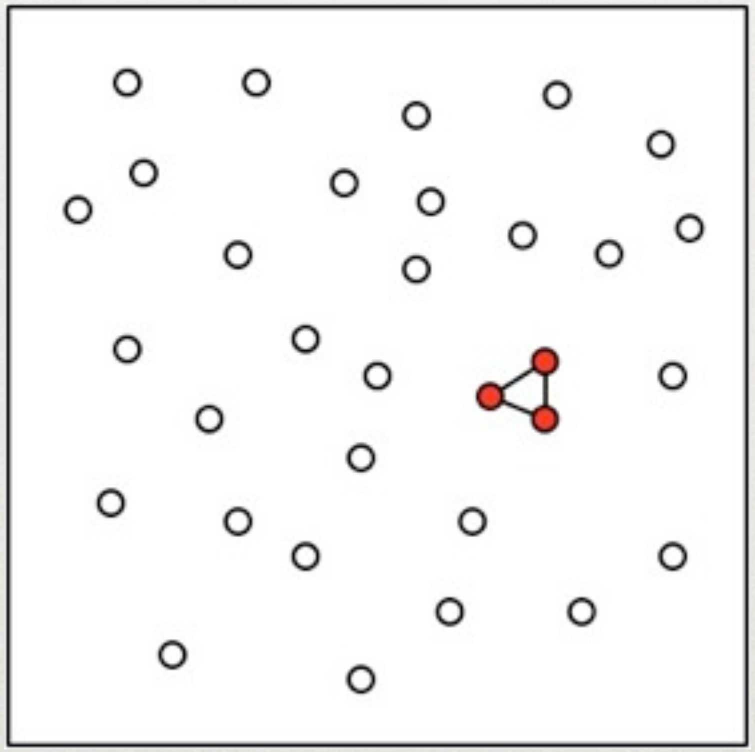

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