The Probabilistic Method

Week 7: Alterations

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CS49/Math59
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Reading Quiz

Which inequalities are valid?

(1) $1/k! < (e/k)^k$

(2) $\binom{n}{k} \leq n^k/k!$

(3) $e^{ck} > 1+ck$

(A) **(1) and (2)**

(B) **(1) and (3)**

(C) **(2) and (3)**

(D) **All are valid**

(E) **None of the above**

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What is $\alpha(G)$?

(A) **clique number of G**

(B) **independence number of G**

(C) **chromatic number of G**

(D) **size of the smallest cycle in G**

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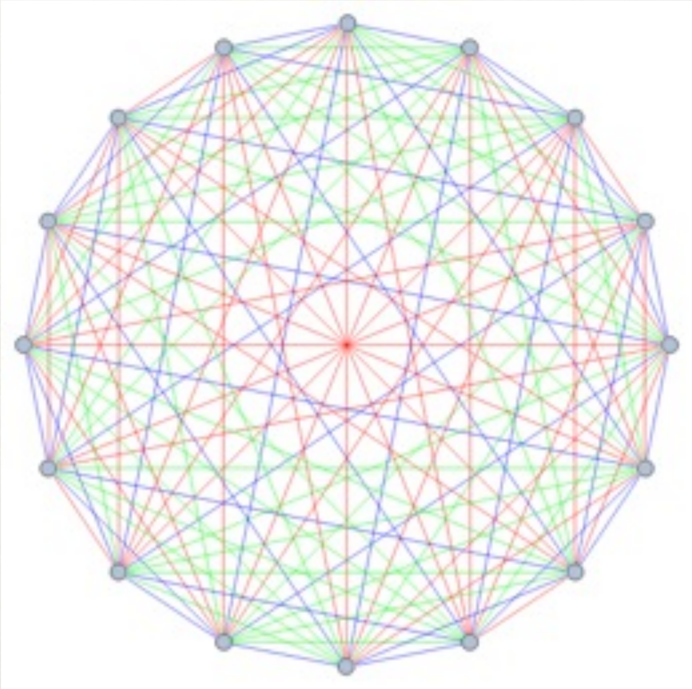
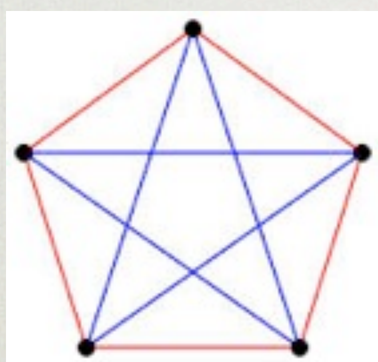


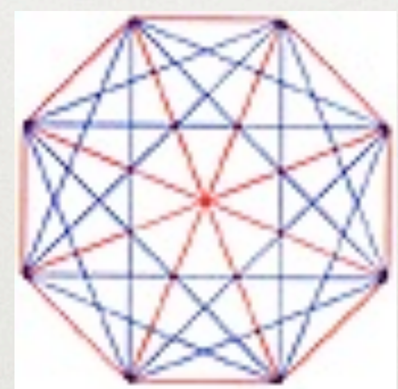
- Define bad events **{BAD_S}**
- **BAD := ∪ BAD_S**
- Compute **Pr[BADS]**
- Calculate **# S**
- **Union Bound: Pr[BAD] ≤ (#S)*Pr[BAD_S]**

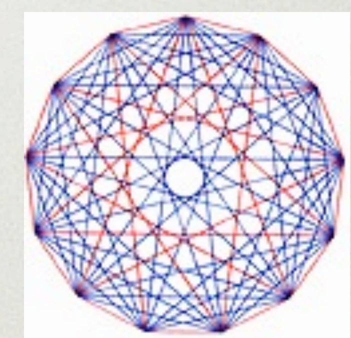
If **Pr[BAD] ≤ 1** then bad events avoided with probability **> 0**

Ramsey Theory

R(k,l) := smallest **n** such that for every two-coloring of K_n, there is red K_k subgraph or a blue K_ℓ subgraph.







Basic Method: If $\binom{n}{k} 2^{1-\binom{k}{2}} < 1$ then **R(k,k) > n**.

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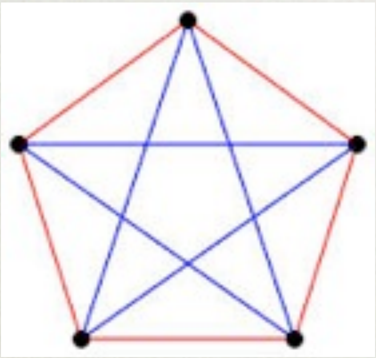

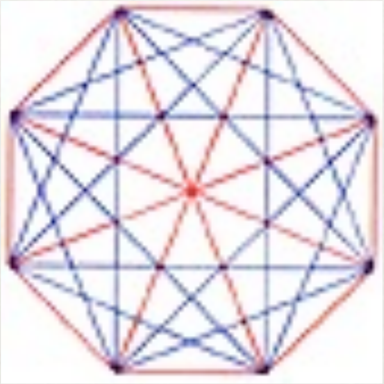

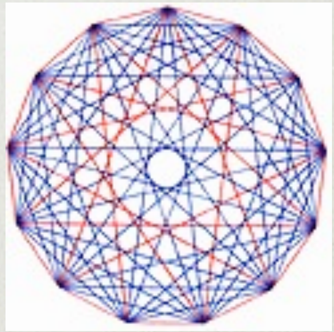

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Alterations: $R(k,k) > n - \binom{n}{k} 2^{1-\binom{k}{2}}$

Maximizing f(n)

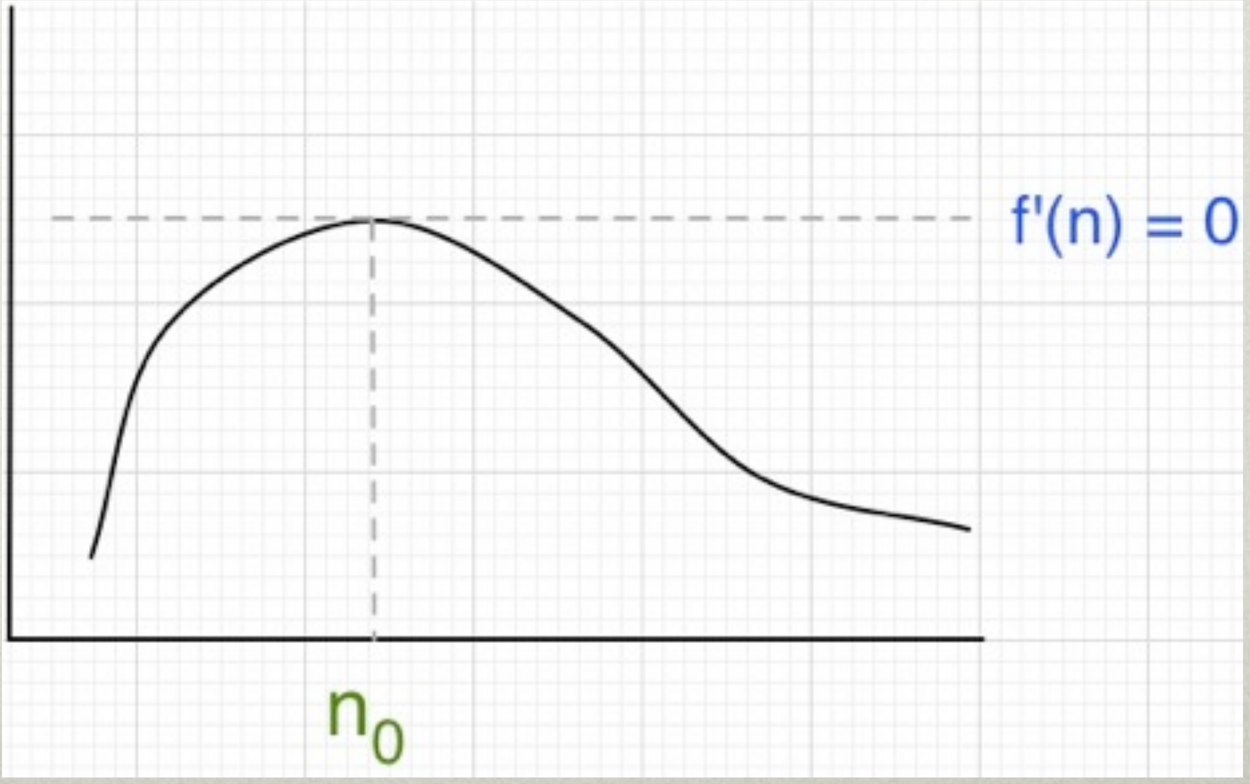
(1) Determine when slope of **$f(n)$** equals **0**:

i.e. find **n_0** so **$f'(n_0) = 0$**

(2) Compute second derivative **$f''(n)$**

(3) If **$f''(n_0) < 0$** then f has local maximum at **n_0**

(4) Compute **$f(n_0)$**



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