

THE PROBABILISTIC METHOD

WEEK 6: EXPECTATION, VARIANCE, AND BEYOND



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CS49/MATH59
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READING QUIZ

What is the following result commonly called:

Theorem: Let **X** be a random variable that takes only nonnegative values. Then for every **$\alpha > 0$** , we have **$\Pr[X \geq \alpha] \leq E[X]/\alpha$** .

- (A) **Chernoff Bound**
- (B) **Markov's Inequality**
- (C) **Chebyshev's Inequality**
- (D) **Wicentowski's Invariant**
- (E) **None of the above**

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VARIANCE

Definition: **$\text{Var}[X] := E[(X - E[X])^2]$**

- measures how far **X** is from expected value
- penalizes large deviations
- standard deviation: **$\sigma = \sqrt{\text{Var}[X]}$**



CLICKER QUESTION

Let **X** have the binomial distribution.

What is **Var[X]**?

- (A) **Var[X] = $(\sqrt{n})/4$**
- (B) **Var[X] = $n/2$**
- (C) **Var[X] = $n/4$**
- (D) **Var[X] = $\sqrt{(2\pi n)} * (n/e)^n$**
- (E) **none of the above**



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There are **300k** workers in Delaware County.

The average income is **\$40k/yr.**

How many can make **\$100k/yr?**

(Use Markov Inequality, choose the most accurate answer)

- (A) **at most 120k workers**
- (B) **at most 196k workers**
- (C) **at most 12k workers**
- (D) **at most 300k workers**
- (E) **none of the above**

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Some of us see the world in terms of expected value. We are very different from the rest of you.

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