

THE PROBABILISTIC METHOD

WEEK 6: EXPECTATION, VARIANCE, AND BEYOND



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CS49/MATH59
FALL 2015

CLICKER QUESTION

Which of the following are not valid inequalities for n !?

- (A) $n! < n^n$
- (B) $n! > \sqrt{2\pi} * n * (n/e)^n$
- (C) $n! < 2\sqrt{\pi n} * (n/e)^n$
- (D) $n! > (n/e)^n$
- (E) **None of the above**

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(C) $n! < 2\sqrt{\pi n} * (n/e)^n$

(D) $n! > (n/e)^n$

(E) **None of the above**

UNBALANCING LIGHTS

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
x_1	+1	-1	+1	+1	+1	-1	+1	+1	-1
x_2	-1	+1	+1	-1	+1	+1	+1	-1	-1
x_3	+1	-1	-1	-1	-1	-1	-1	-1	-1
x_4	-1	-1	+1	+1	-1	+1	+1	-1	-1
x_5	-1	-1	-1	+1	+1	-1	-1	-1	-1
x_6	-1	-1	-1	-1	+1	+1	+1	+1	-1
x_7	+1	+1	+1	+1	-1	-1	-1	-1	-1
x_8	-1	+1	+1	+1	+1	-1	+1	+1	-1
x_9	+1	+1	-1	+1	-1	+1	+1	+1	+1

UNBALANCING LIGHTS

	y ₁	y ₂	y ₃	y ₄	y ₅	y ₆	y ₇	y ₈	y ₉
x ₁	+1	-1	+1	+1	+1	-1	+1	+1	-1
x ₂	-1	+1	+1	-1	+1	+1	+1	-1	-1
x ₃	+1	-1	-1	-1	-1	-1	-1	-1	-1
x ₄	-1	-1	+1	+1	-1	+1	+1	-1	-1
x ₅	-1	-1	-1	+1	+1	-1	-1	-1	-1
x ₆	-1	-1	-1	-1	+1	+1	+1	+1	-1
x ₇	+1	+1	+1	+1	-1	-1	-1	-1	-1
x ₈	-1	+1	+1	+1	+1	-1	+1	+1	-1
x ₉	+1	+1	-1	+1	-1	+1	+1	+1	+1

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	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
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x_2	-1	+1	+1	-1	+1	+1	+1	-1	-1
x_3	+1	-1	-1	-1	-1	-1	-1	-1	-1
x_4	-1	-1	+1	+1	-1	+1	+1	-1	-1
x_5	-1	-1	-1	+1	+1	-1	-1	-1	-1
x_6	+1	+1	+1	+1	-1	-1	-1	-1	+1
x_7	+1	+1	+1	+1	-1	-1	-1	-1	-1
x_8	-1	+1	+1	+1	+1	-1	+1	+1	-1
x_9	+1	+1	-1	+1	-1	+1	+1	+1	+1

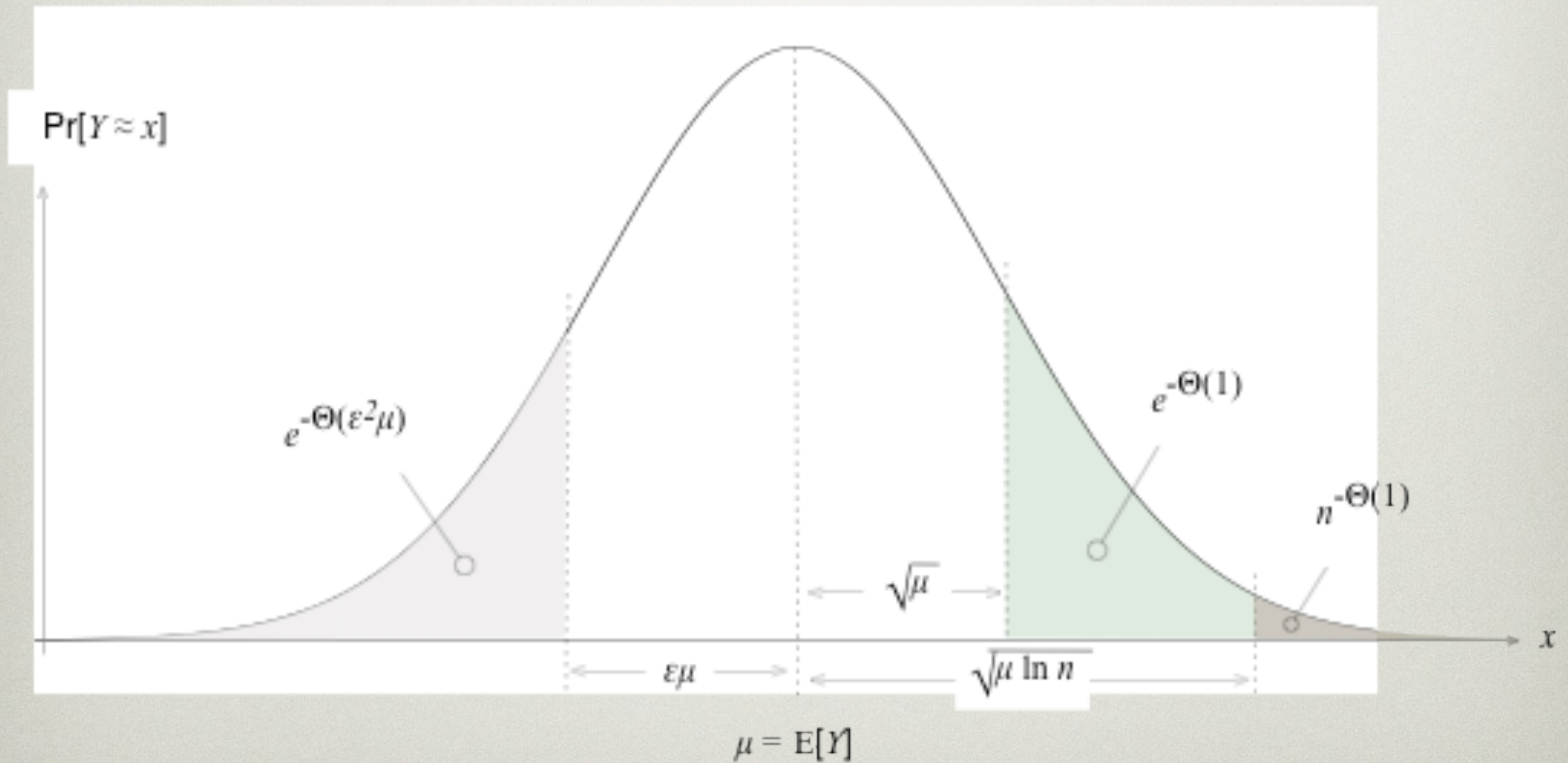
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	y ₁	y ₂	y ₃	y ₄	y ₅	y ₆	y ₇	y ₈	y ₉
x ₁	+1	-1	+1	+1	+1	-1	+1	+1	-1
x ₂	-1	+1	+1	-1	+1	+1	+1	-1	-1
x ₃	+1	-1	-1	-1	-1	-1	-1	-1	-1
x ₄	-1	-1	+1	+1	-1	+1	+1	-1	-1
x ₅	-1	-1	-1	+1	+1	-1	-1	-1	-1
x ₆	+1	+1	+1	+1	-1	-1	-1	-1	+1
x ₇	+1	+1	+1	+1	-1	-1	-1	-1	-1
x ₈	-1	+1	+1	+1	+1	-1	+1	+1	-1
x ₉	+1	+1	-1	+1	-1	+1	+1	+1	+1

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x_3	+1	-1	-1	-1	-1	-1	-1	+1	-1
x_4	-1	-1	+1	+1	-1	+1	+1	+1	-1
x_5	-1	-1	-1	+1	+1	-1	-1	+1	-1
x_6	+1	+1	+1	+1	-1	-1	-1	+1	+1
x_7	+1	+1	+1	+1	-1	-1	-1	+1	-1
x_8	-1	+1	+1	+1	+1	-1	+1	-1	-1
x_9	+1	+1	-1	+1	-1	+1	+1	-1	+1

BINOMIAL DISTRIBUTION



VARIANCE

Definition: **$\text{Var}[X] := E[(X - E[X])^2]$**

- measures how far **X** is from expected value
- penalizes large deviations
- standard deviation: **$\sigma = \sqrt{\text{Var}[X]}$**



CLICKER QUESTION

Let **X** be a **p**-biased coin:

- **$\Pr[X=1] = p$**
- **$\Pr[X=0] = 1-p$**

What is **$\text{Var}[X]$** ?

- (A) **$\text{Var}[X] = pq$**
- (B) **$\text{Var}[X] = 1/2$**
- (C) **$\text{Var}[X] = p$**
- (D) **$\text{Var}[X] = q^2p + p^2q$**
- (E) **multiple answers correct**

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- (D) **$\text{Var}[X] = q^2p + p^2q$**
- (E) **multiple answers correct**

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Some of us see the world in terms of expected value. We are very different from the rest of you.

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