The Probabilistic Method

Week 5: Linearity Of Expectation

Joshua Brody

CS49/Math59

Fall 2015

Clicker Question

Uniformly two-color edges of K_n. How many monochromatic subgraphs K_a should you expect to get?

(A) $\binom{n}{a}2^{-\binom{a}{2}}$

(B) $2\binom{n}{a}2^{-\binom{a}{2}}$

(C) $\binom{n}{2a}2^{1-\binom{a}{2}}$

(D) $n^a 2^{-\binom{a}{2}}$

(E) **None of the above**

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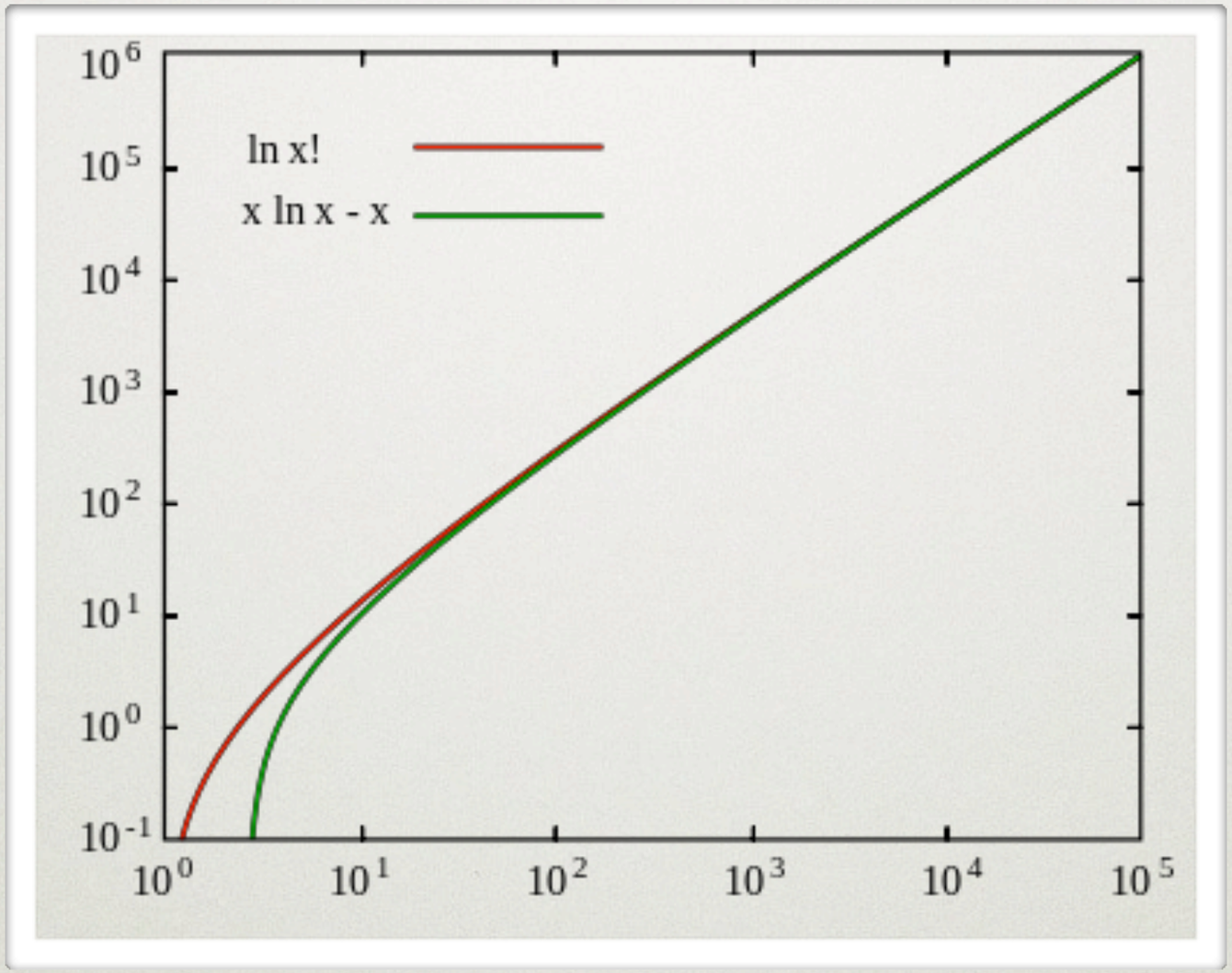
Stirling's Approximation



log(n!) vs **log((n/e)^n)**

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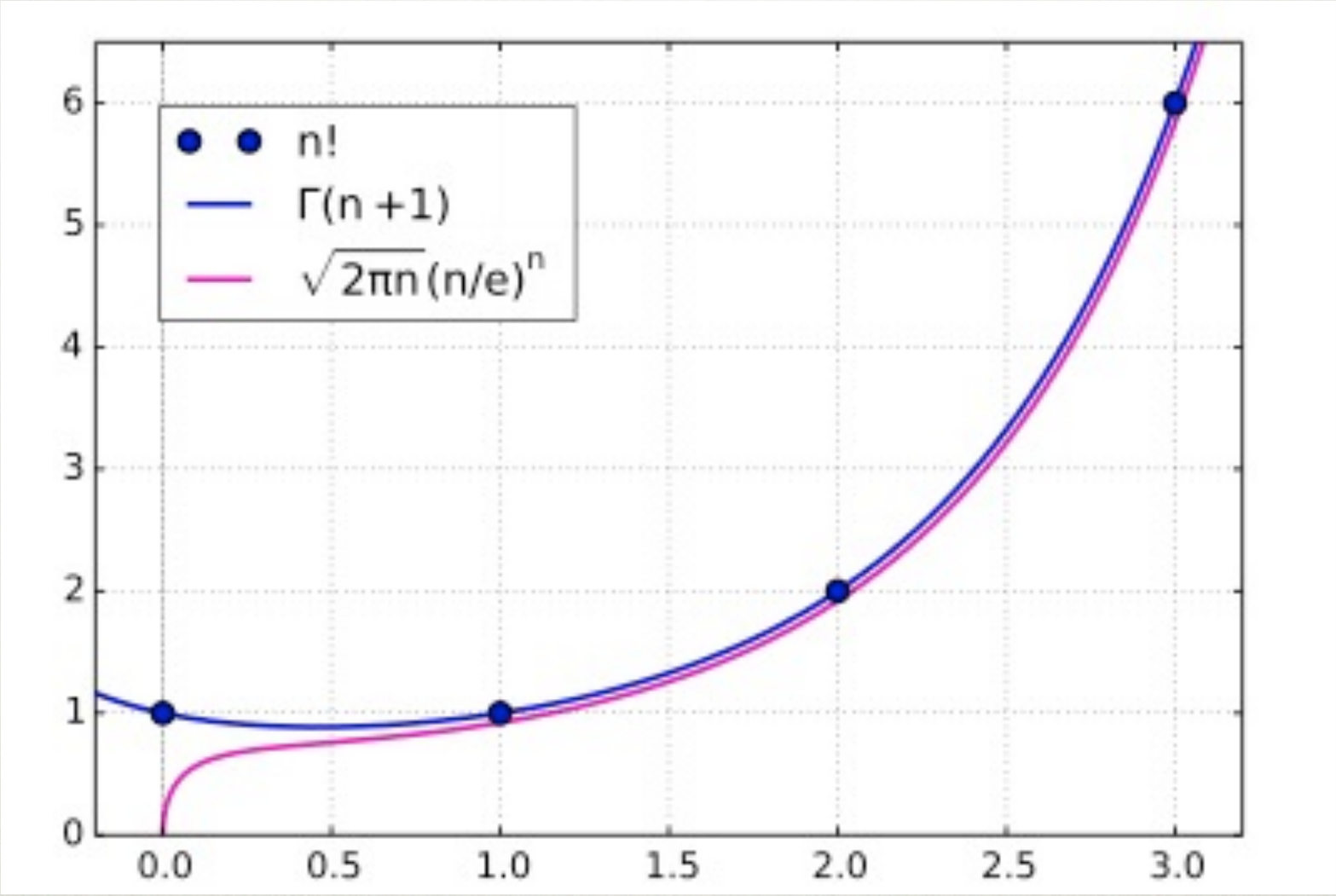

n! vs **√(2Πn)(n/e)ⁿ**

Unbalancing Lights

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
x_1	+1	-1	+1	+1	+1	-1	+1	+1	-1
x_2	-1	+1	+1	-1	+1	+1	+1	-1	-1
x_3	+1	-1	-1	-1	-1	-1	-1	-1	-1
x_4	-1	-1	+1	+1	-1	+1	+1	-1	-1
x_5	-1	-1	-1	+1	+1	-1	-1	-1	-1
x_6	-1	-1	-1	-1	+1	+1	+1	+1	-1
x_7	+1	+1	+1	+1	-1	-1	-1	-1	-1
x_8	-1	+1	+1	+1	+1	-1	+1	+1	-1
x_9	+1	+1	-1	+1	-1	+1	+1	+1	+1

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x_4	-1	-1	+1	+1	-1	+1	+1	-1	-1
x_5	-1	-1	-1	+1	+1	-1	-1	-1	-1
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x_5	-1	-1	-1	+1	+1	-1	-1	+1	-1
x_6	+1	+1	+1	+1	-1	-1	-1	+1	+1
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x_8	-1	+1	+1	+1	+1	-1	+1	-1	-1
x_9	+1	+1	-1	+1	-1	+1	+1	-1	+1

Clicker Question

Flip a fair coin **n** times. What is **Pr[#heads = n/2]**?

(A) $\sqrt{(2/\pi)}(1+o(1))$

(B) $\sqrt{(2\pi/n)}(1+o(1))$

(C) $2^n\sqrt{(2/\pi n)}$

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(E) **None of the above**

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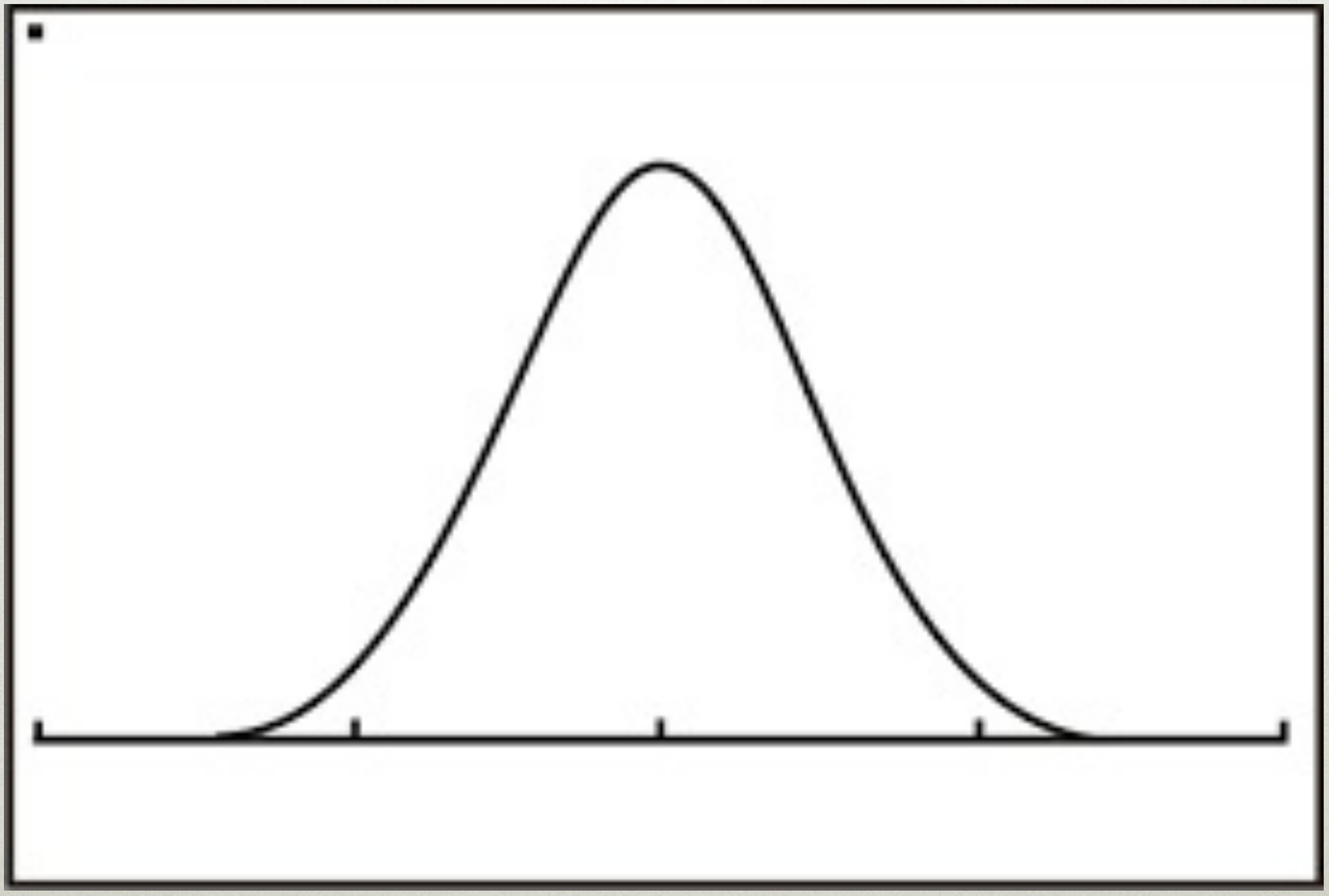
Binomial Distribution



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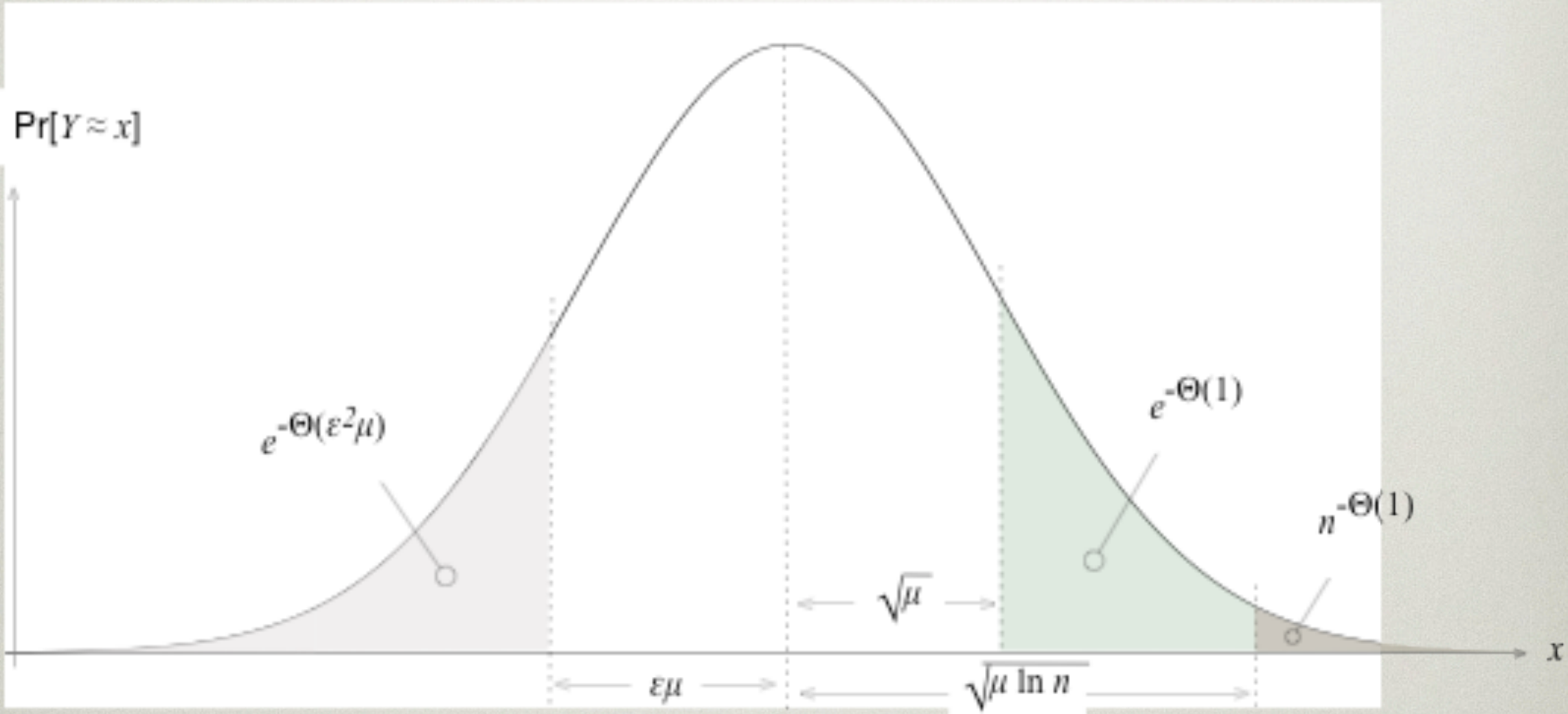

$\mu = \mathrm{E}[Y]$

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