The Probabilistic Method

Week 3: Asymptotic Analysis

Joshua Brody

CS49/Math59

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Clicker Question

Let **$f(n) = n^2 + 17n$** and **$g(n) = 10n^2 - n/1000$**.

Which answer most accurately represents the relationship between f and g?

(A) **$f(n) = \Theta(g(n))$**

(B) **$f(n) = O(g(n))$**

(C) **$f(n) = \Omega(g(n))$**

(D) **$f(n) = o(g(n))$**

(E) **$f(n) = \omega(g(n))$**

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Suppose that **f = o(g).** Which of the following values for **f(n)** and **g(n)** are possible?

(A) **f(n) = n, g(n) = 10n**

(B) **$f(n) = n^3$, $g(n) = 100n^2$**

(C) **$f(n) = n \log n$, $g(n) = 10n^2$**

(D) **$f(n) = 10n$, $g(n) = 2^{2 + \log(n)}$**

(E) **multiple answers possible.**

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(E) **multiple answers possible.**

Transitivity of Asymptotic Notations

Facts: Let **f,g,h** be functions.

(1) if **f = O(g)** and **g = O(h)**, then **f = O(h)**.

(2) If **f = Ω(g)** and **g = Ω(h)**, then **f = Ω(h)**.

(3) If **f = Θ(g)** and **g = Θ(h)**, then **f = Θ(h)**.

(4) If **f = o(g)** and **g = o(h)**, then **f = o(h)**.

(5) If **f = ω(g)** and **g = ω(h)**, then **f = ω(h)**.

(6) If **f ~ g** and **g ~ h**, then **f ~ h**.

Asymptotic Notation

- **$f(n) = O(g(n))$** if there exists constants **$c, n_0 > 0$** such that for all **$n \geq n_0$**,

$$f(n) \leq cg(n)$$

- **$f(n) = \Omega(g(n))$** if there exists constants **$c, n_0 > 0$** such that for all **$n \geq n_0$**,

$$f(n) \geq cg(n)$$

- **$f(n) = \Theta(g(n))$** if there are constants **$c_1, c_2, n_0 > 0$** such that for all **$n \geq n_0$**,

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

Asymptotic Notation

- **f(n) = o(g(n))** if for all constants **c>0** there exists **n_0 > 0** such that for all **n≥n_0**,

  f(n) < cg(n)

- **f(n) = ω(g(n))** if for all constants **c>0** there exists **n_0>0** such that for all **n≥n_0**,

  f(n) > cg(n)

Asymptotic Properties

Facts: Let **f,g,h** be functions.

(1) if **f = O(g)** and **g = o(h)**, then **f = o(h)**.

(2) If **f = O(h)** and **g = O(h)**, then **f+g = O(h)**.

(3) If $f_1,\ldots,f_k = O(h)$ then $f_1+\ldots+f_k = O(h)$ as long as **k** is constant.

(4) If $f_1 = O(g_1)$ and $f_2 = O(g_2)$, then $f_1+f_2 = O(g_1+g_2)$.

(5) If $f_1 = O(g_1)$ **and** $f_2 = O(g_2)$ **then** $f_1f_2 = O(g_1g_2)$.

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