# The Probabilistic Method

## Week 2: Independence, Random Variables, Asymptopia

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CS49/Math59
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# Clicker Question

Suppose that

- $\Omega = \{1, \ldots, 6\} \times \{1, \ldots, 6\}$,
- $X(s,t) = s$, and
- $Y(s,t) = t$.

What is "**X == Y**"?

(A) **random variable, indicating that X must equal Y**

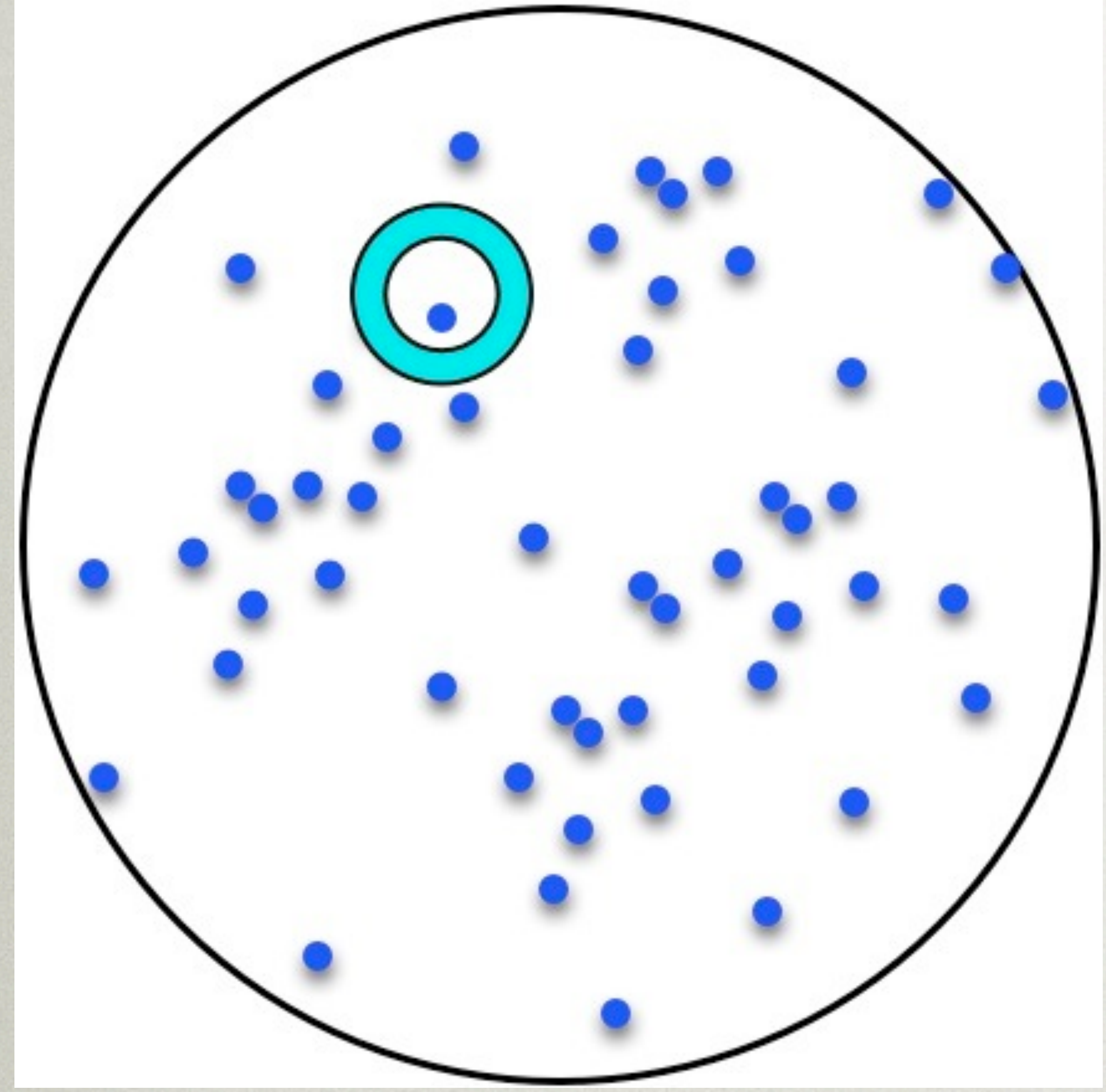
(B) **{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}**

(C) **event that X equals Y**

(D) **multiple answers correct**

(E) **none of the above**

# The Washer Problem



**650** points placed in circle of radius **16**.

Show you can place a washer (inner radius **2**, outer radius **3**) so it covers **10** points.

# Washer Problem Analysis

**Solution:** randomly place center of washer in circle of radius 19.

**helpful question:** *how does each point get covered?*

**unhelpful questions:**

*how are points spread out in circle?*

*what's the worst-case situation?*

# Washer Problem Analysis

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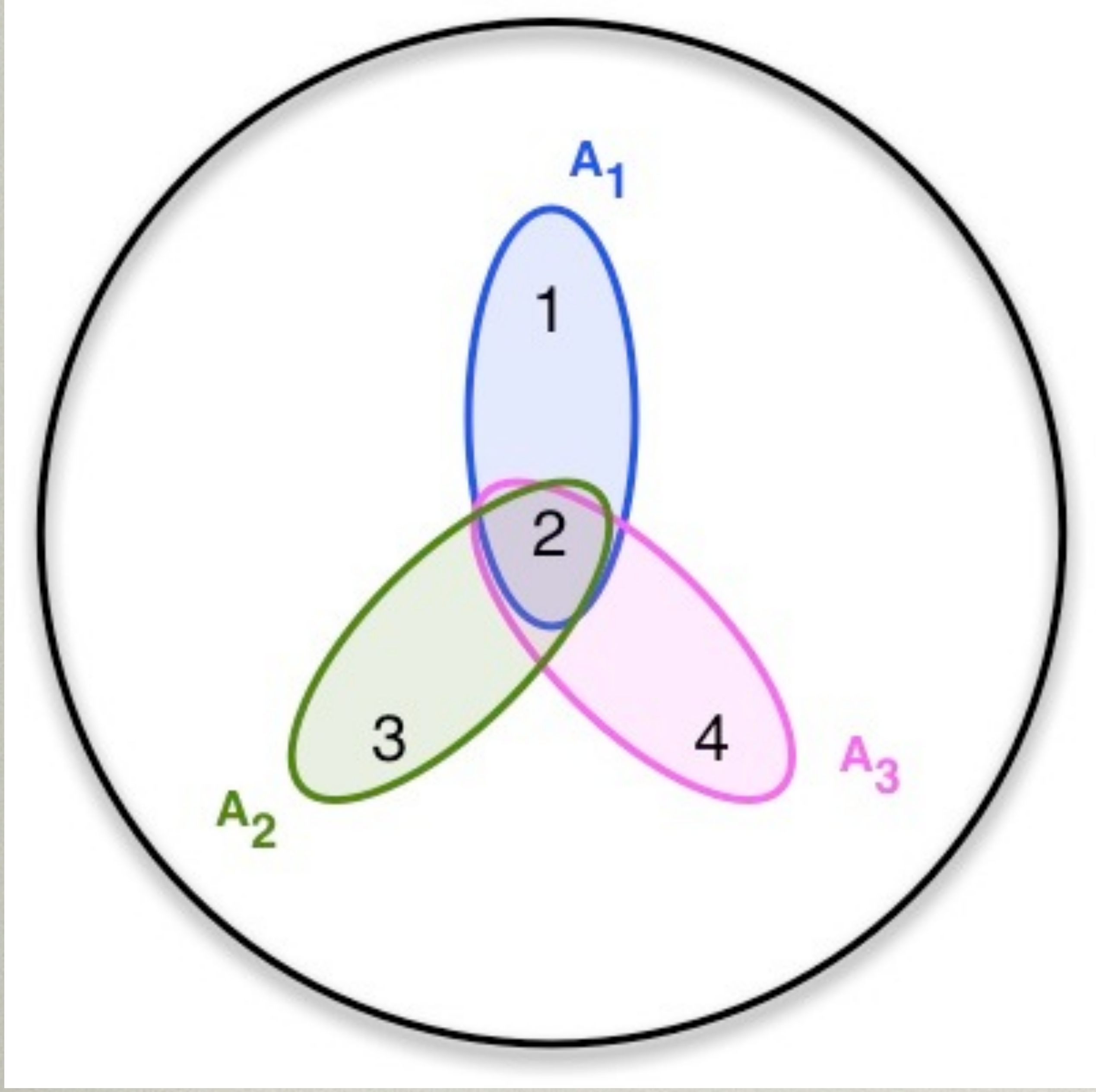
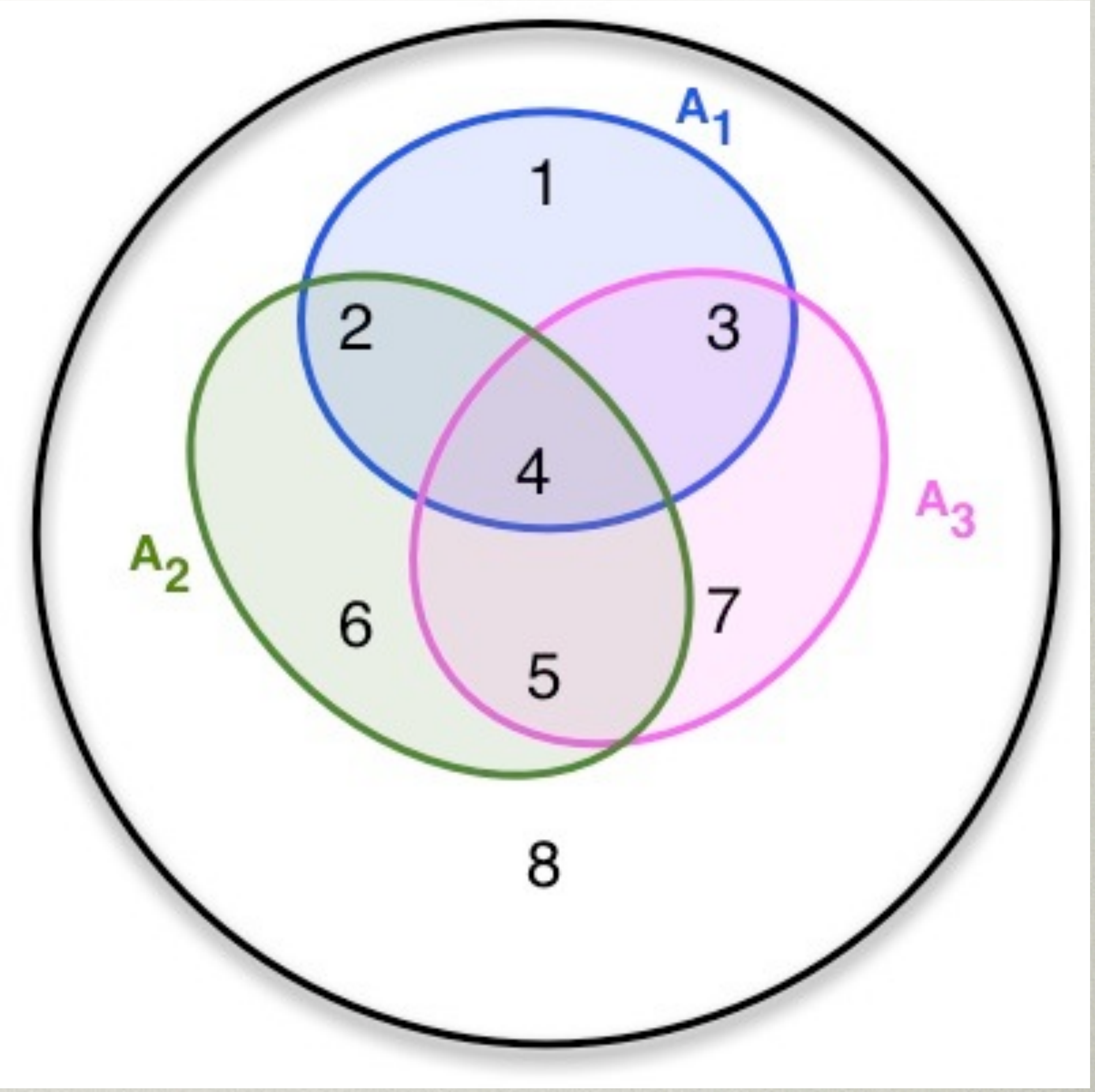
*What is the intuition behind using randomness? We want a solution that does not have structure, because structure can be used against us. Randomness is a way to get such a solution.*

# Independent Events



# Clicker Question

Let **P** be uniform on **{1, 2, ..., 10}**.

Let **A = {2, 3, 5, 7}**, **B = {1, 3, 5, 7, 9}**, **C={1,2,3,4}.**

Which events are independent?

(A) **A and B**

(B) **A and C**

(C) **B and C**

(D) **A, B, and C**

(E) **none**

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Let $X_1$, $X_2$, $X_3$ be fair coins. In which of the following circumstances are $X_1$, $X_2$, $X_3$ 2-wise independent?

(A) $X_1$, $X_2$, $X_3$ mutually independent.

(B) $X_1$, $X_2$ independent; $X_3 = X_1 \oplus X_2$

(C) $X_3 = X_1$ w/prob 1/3, $X_3 = X_2$ w/prob 2/3

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