The Probabilistic Method

Week 2: Independence, Random Variables, Asymptopia

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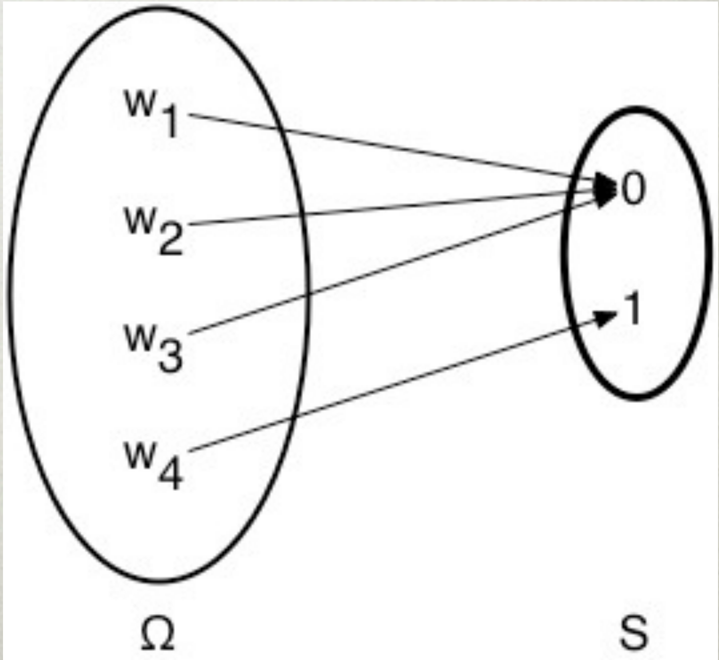
CS49/Math59
Fall 2015

Random Variables

Let **P** be a probability distribution on **Ω.**

A random variable **X** is a function **X : Ω → S.**

- **"X = *s*"** is the event **$\{w \in \Omega : X(w) = s\}$**
- **$P[X{=}s] = \sum_{w\,:\,X(w)\,=\,s} P(w)$**
- distribution **P_X** of **X**: **$P_X(s) = P[X{=}s]$**
- **X is real-valued if $S \subseteq \mathbb{R}$**



Clicker Question

Let **P** be uniform on **Ω = {1, 2, ..., 100}**.

Define random variable **X** by **X(w) = w (mod 7)**

What is **$P_X(4)$**?

(A) **0.01**

(B) **0.14**

(C) **1/6**

(D) **1/7**

(E) **none of the above**

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How are these random variables *related*?

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(A) X_1, X_2, X_3 mutually independent.

(B) X_1, X_2 independent; $X_3 = X_1 \oplus X_2$

(C) $X_3 = X_1$ w/prob 1/3, $X_3 = X_2$ w/prob 2/3

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