The Probabilistic Method

Week 13: P, NP, SAT

Joshua Brody
CS49/Math59
Fall 2015

Quiz

Which of the following languages are known to be in P?

(A) **THREE-COLOR = {G = (V,E): G can be colored using three colors so that no edge is monochromatic}**

(B) **BIPARTITE = {G = (V,E): G is bipartite}**

(C) **PRIMES = {integers n: n is a prime number}**

(D) **FACTORING = {(n,k) : n has factor d s.t. 1 < d < k}**

(E) **Multiple Answers Correct**

Quiz

Which of the following languages are known to be in P?

(A) THREE-COLOR = {G = (V,E): G can be colored using three colors so that no edge is monochromatic}

(B) BIPARTITE = {G = (V,E): G is bipartite}

(C) PRIMES = {integers n: n is a prime number}

(D) FACTORING = {(n,k) : n has factor d s.t. 1 < d < k}

(E) Multiple Answers Correct

Clicker Question

Give a representation of two arbitrarily large integers **(n,k)** as a single bit string.

Clicker Question

Give a representation of two arbitrarily large integers **(n,k)** as a single bit string.

How many bits are in the representation? *(ignore ceiling function)*

(A) **log(n) + log(k)**

(B) **max[log(n), log(k)]**

(C) **3log(nk)**

(D) **2max[log(n), log(k)]**

(E) **None of the above**

Clicker Question

Give a representation of two arbitrarily large integers **(n,k)** as a single bit string.

How many bits are in the representation? *(ignore ceiling function)*

(A) log(n) + log(k)

(B) max[log(n), log(k)]

(C) 3log(nk)

(D) 2max[log(n), log(k)]

(other answers possible)

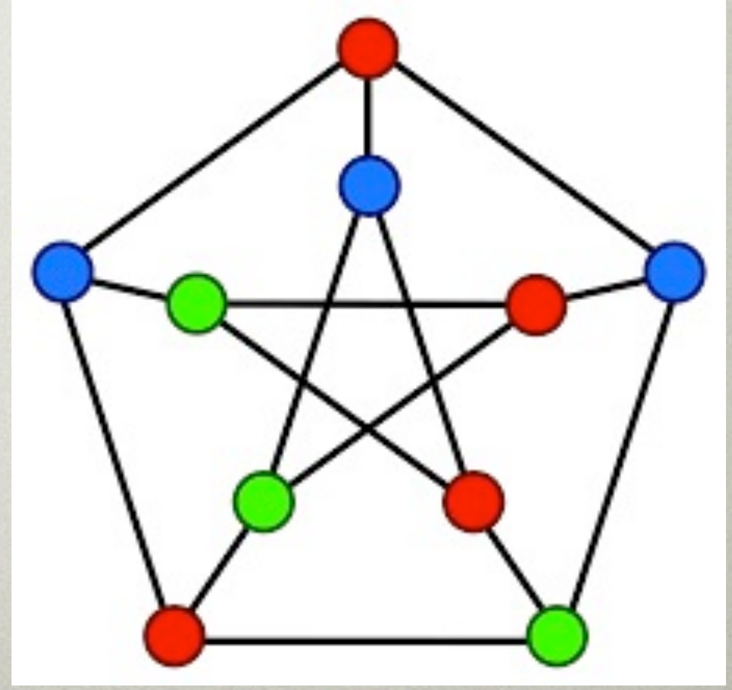
(E) None of the above

Polynomial Time Verifiers

V is an *efficient verifier* for a decision problem **L** if:

(1) **V** is a polynomial time algorithm that takes two inputs: **x** and **w**

(2) **$x \in L$** iff there is w such that **$\text{length}(w) = \text{length}(x)^{O(1)}$** and **V(x,w) = YES**



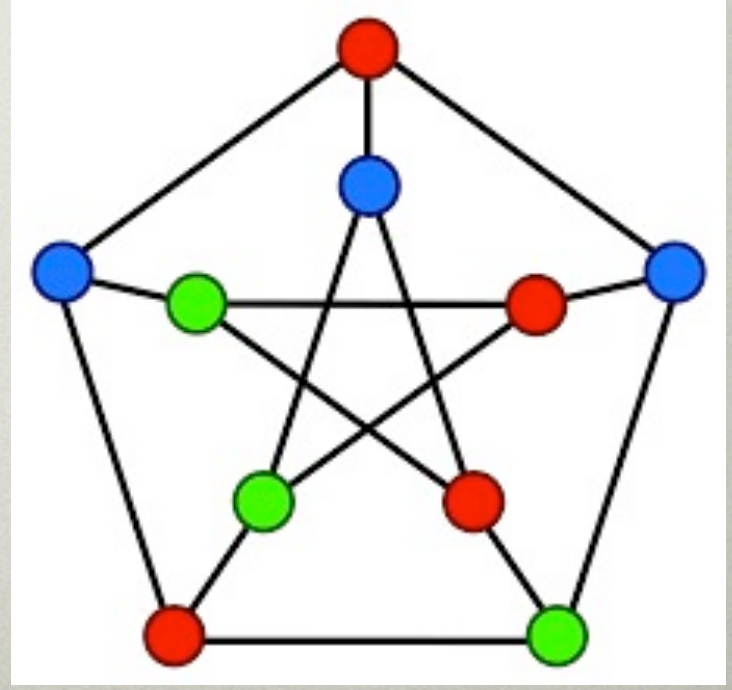
Polynomial Time Verifiers

V is an *efficient verifier* for a decision problem **L** if:

(1) **V** is a polynomial time algorithm that takes two inputs: **x** and **w**

(2) **$x \in L$** iff there is w such that **$\text{length}(w) = \text{length}(x)^{O(1)}$** and **V(x,w) = YES**

NP:= set of languages **verifiable** in polynomial time



Hard Problems?

Problems for which no **polytime algorithm** is known:

- **INDEPENDENT-SET**: Given **$G = (V,E)$** and integer **k**, is there an *independent set* of size at least **k**?
- **VERTEX-COVER**: Given **$G = (V,E)$** and integer **k**, is there a *vertex cover* of size at most **k**?
- **FACTORING**: Given integers **(n,k)**, does **n** have a *factor* **$1 < d < k$**?
- **SUBSET-SUM**: Given set of integers **$A = \{a_1, a_2, \ldots, a_n\}$** is there a *subset* **$S \subseteq A$** that *sums to zero*?

Clicker Question

Consider the following SAT instance: $c_1 \wedge c_2 \wedge c_3 \wedge c_4$

for the following clauses:

$c_1 = (x_1 \vee x_2 \vee \neg x_3)$ $c_2 = (\neg x_1 \vee x_3 \vee x_4)$

$c_3 = (\neg x_2 \vee \neg x_3 \vee \neg x_4)$ $c_4 = (x_2 \vee \neg x_3 \vee x_4)$

Which of the following are satisfying assignments?

(A) $(x_1, x_2, x_3, x_4) = (F, F, F, F)$

(B) (T, T, T, T)

(C) (T, F, T, F)

(D) (F, T, F, F)

(E) Multiple Answers Correct

Clicker Question

Consider the following SAT instance: $c_1 \wedge c_2 \wedge c_3 \wedge c_4$

for the following clauses:

$c_1 = (x_1 \vee x_2 \vee \neg x_3)$ $c_2 = (\neg x_1 \vee x_3 \vee x_4)$

$c_3 = (\neg x_2 \vee \neg x_3 \vee \neg x_4)$ $c_4 = (x_2 \vee \neg x_3 \vee x_4)$

Which of the following are satisfying assignments?

(A) $(x_1, x_2, x_3, x_4) = (F, F, F, F)$

(B) (T, T, T, T)

(C) (T, F, T, F)

(D) (F, T, F, F)

(E) Multiple Answers Correct

Clicker Question

What is the input length of SAT?

(how would you efficiently encode a SAT input?)

(A) **$O(n+m)$**

(B) **$O(nm)$**

(C) **$O(n + m\log(n))$**

(D) **$O(2^n)$**

(E) **None of the above**

Clicker Question

What is the input length of SAT?

(how would you efficiently encode a SAT input?)

(A) O(n+m)

(B) O(nm)

(C) O(n + mlog(n))

(D) $O(2^n)$

(E) None of the above

Clicker Question

Design an algorithm recognizing SAT.

What is its runtime?

(A) $O(2^n m)$

(B) $O(n^m)$

(C) $O(nm)$

(D) $O(n^2 m)$

(E) $O(n + m)$

Clicker Question

Design an algorithm recognizing SAT.

What is its runtime?

(A) $O(2^n m)$

(B) $O(n^m)$

(C) $O(nm)$

(D) $O(n^2m)$

(E) $O(n^+ m)$

The Probabilistic Method