The Probabilistic Method

Week 12: P vs NP

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CS49/Math59

Fall 2015

Reading Quiz

Which of the following is not an optimization problem?

(A) minimum spanning tree

(B) primality testing

(C) shortest path

(D) maximum clique

(E) Multiple Answers Correct

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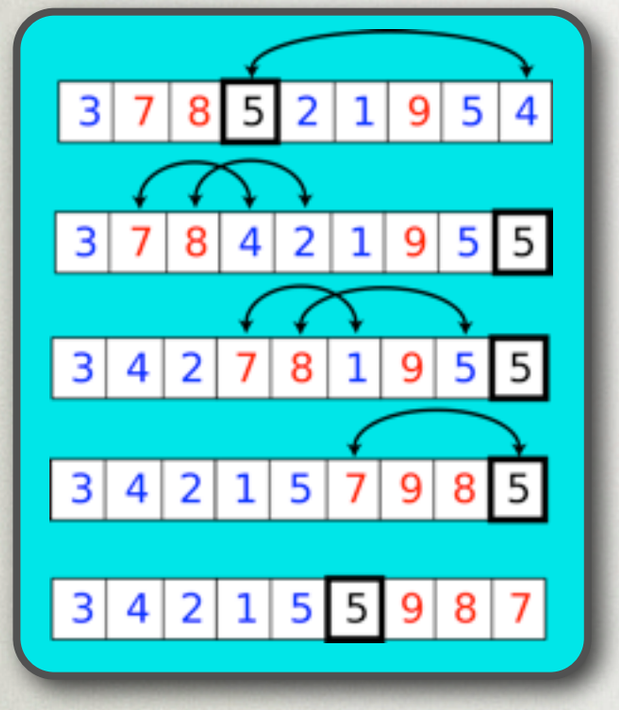
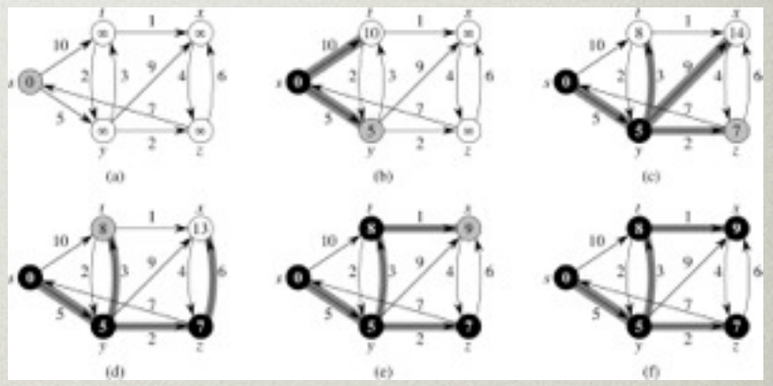
What are Polynomial time Algorithms?





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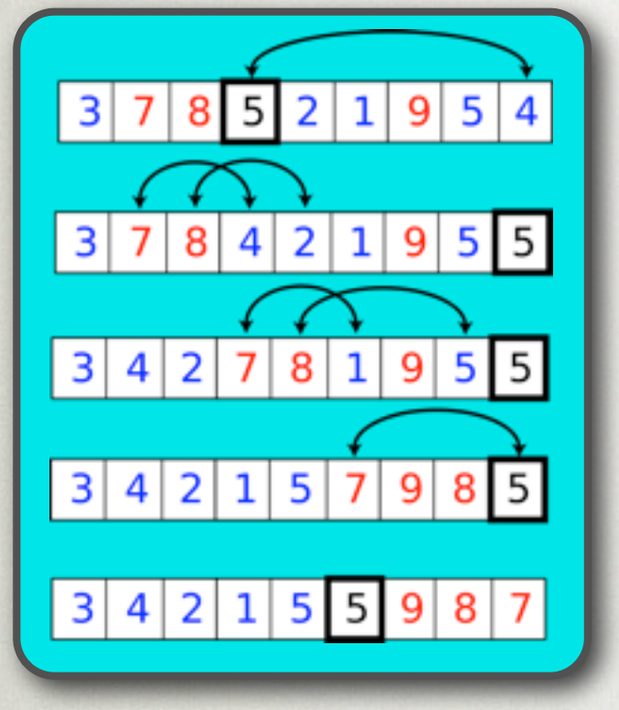
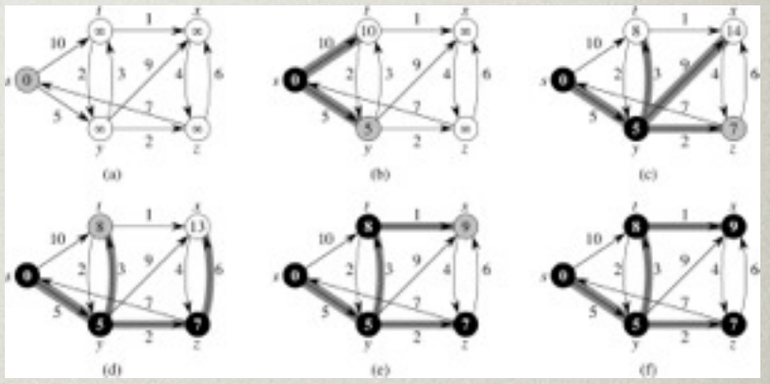


Sorting list of elements:

Dijkstra's Algorithm (shortest paths)



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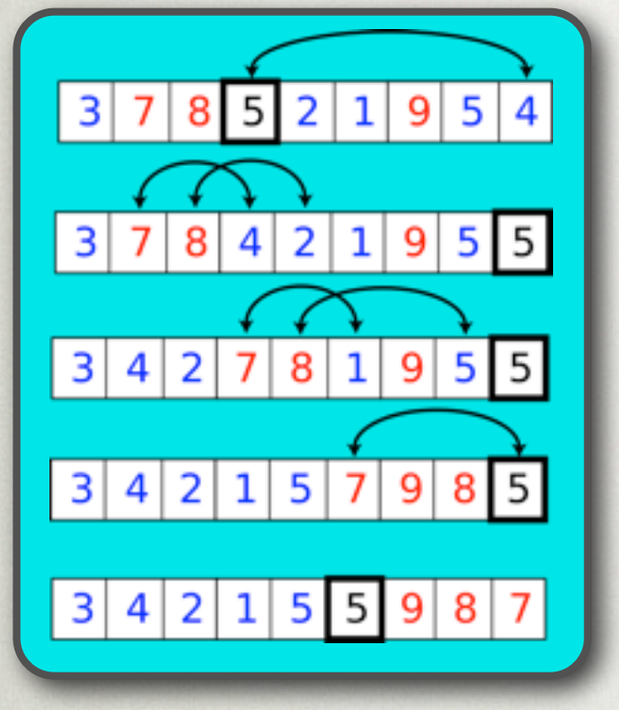
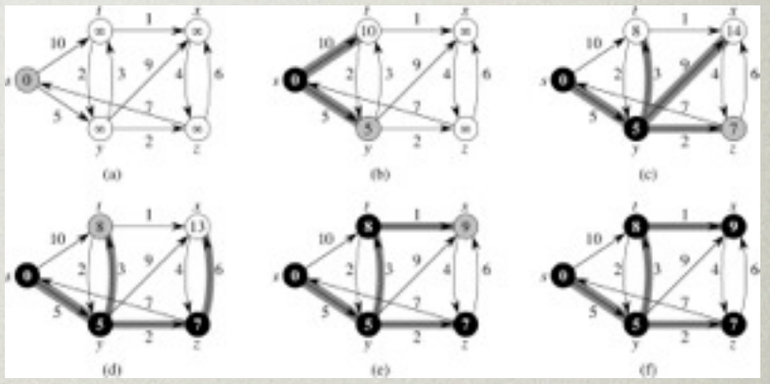


Sorting list of elements:

- **O(n log n)** time possible
- **n**: length of list

Dijkstra's Algorithm (shortest paths)

- **O(m log n)** time
- **n**: number of vertices
- **m**: number of edges



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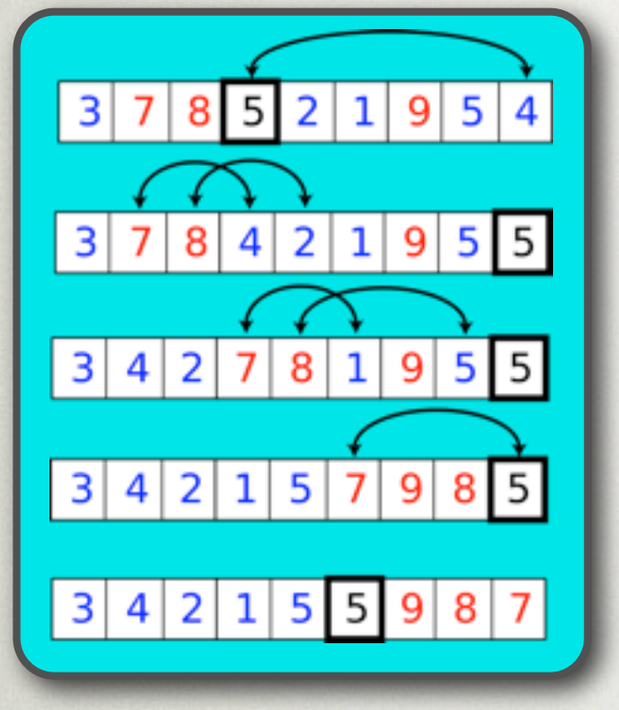

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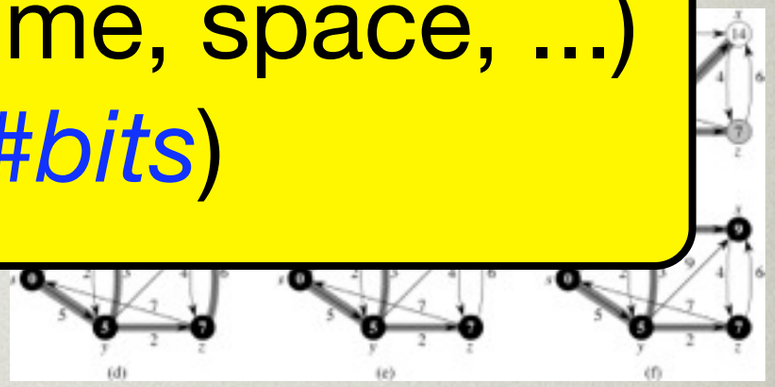
Default: algorithm efficiency (runtime, space, …) is measured in **size of input.** (i.e. *#bits*)



- **m**: number of edges

Clicker Question

Algorithm **A** takes as input a graph **G** on *n vertices*.

What is the size of the input? (use reasonable encoding of G)

(A) $O(n^3)$

(B) $O(n^2)$

(C) $O(2^n)$

(D) $O(\log n)$

(E) None of the above

Clicker Question

Algorithm **A** takes as input a graph **G** on *n vertices*.

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(D) O(log n)

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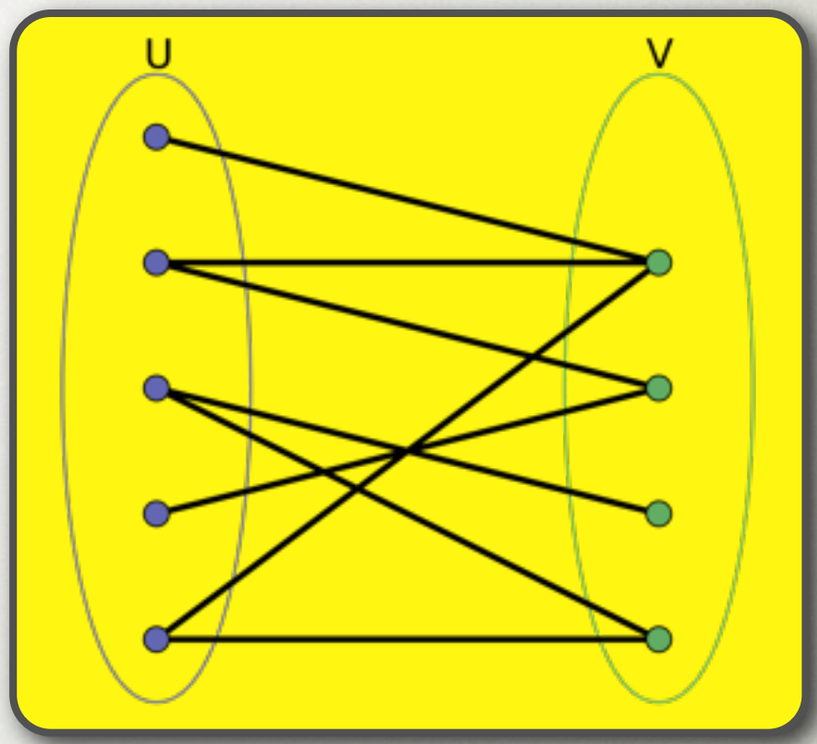
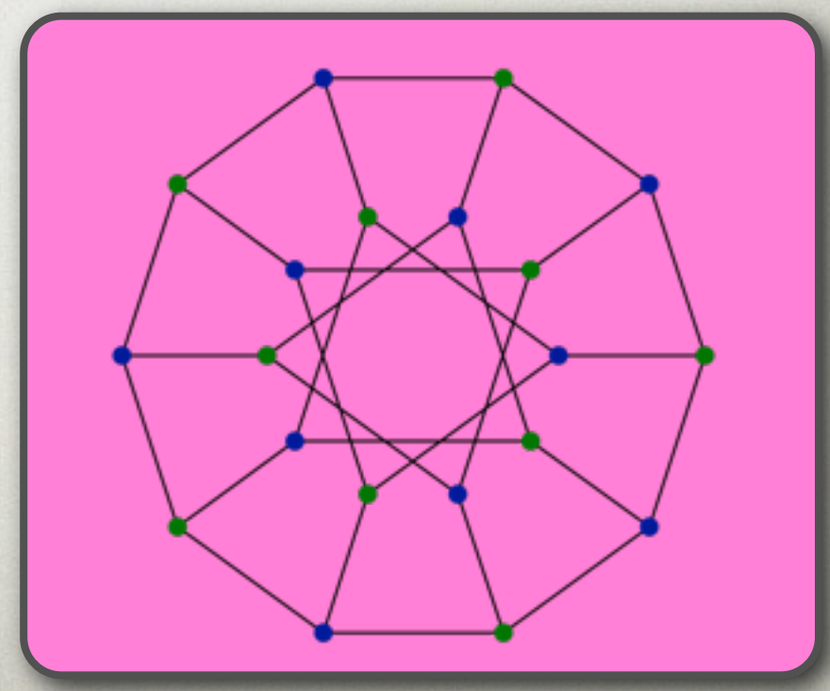
Testing Bipartiteness

```
testBipartite(G = (V,E)):
 initialize color[1...n] to NONE
 // color nodes BLUE or PINK
 color[1] = BLUE
 stack.push(1)
 while(stack not empty) {
  u = stack.pop()
  for each neighbor v of u
  if(color[v] == NONE) {
    color[v] = opposite of u
  } else if (u,v same color) {
    return NO
  }
 } // end while loop
 return YES
}
```





Clicker Question

What is the running time of **testBipartite**?

(A) O(n log n)

(B) $O(n^2 \log n)$

(C) O(n+m)

(D) $O(2^n)$

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Clicker Question

Design and analyze an algorithm that recognizes **THREE-COLOR**. What is its running time?

(A) O(n log n)

(B) $O(2^n)$

(C) O(n+m)

(D) $O(m3^n)$

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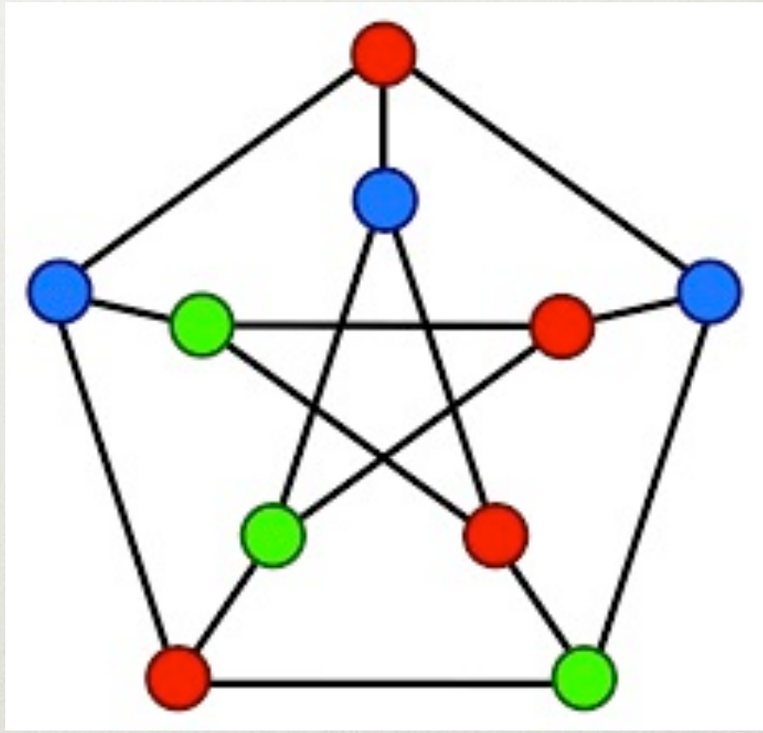

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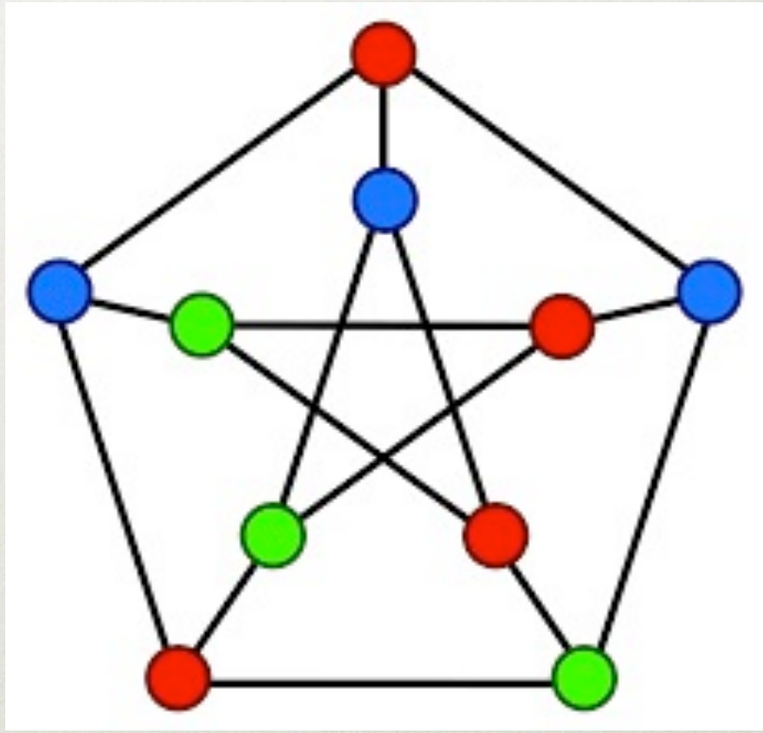

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