The Probabilistic Method

Week 10: Applications

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CS49/Math59

Fall 2015

Reading Quiz

What is the Hamming Distance between two n-bit strings x,y?

(A) dist(x,y) = #i such that x_i != y_i

(B) dist(x,y) = sqrt($\sum_i |x_i - y_i|$)

(C) dist(x,y) = $\max_i |x_i - y_i|$

(D) multiple answers correct

(E) none of the above

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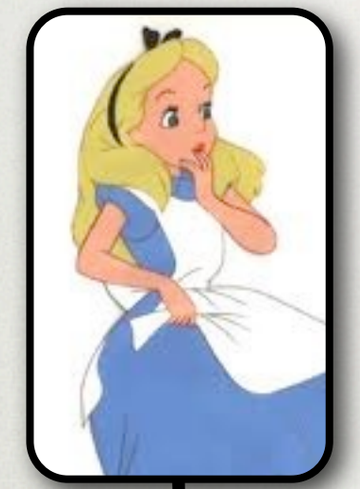
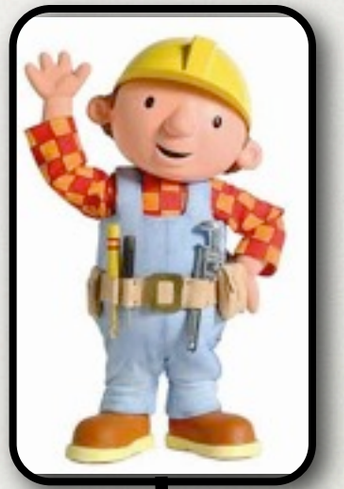
Coding Theory

[Shannon 48]





00010101

Coding Theory

[Shannon 48]

Problem:

- **p**-fraction of bits are corrupted
- Bob doesn’t know which bits get corrupted.

Coding Theory

[Shannon 48]

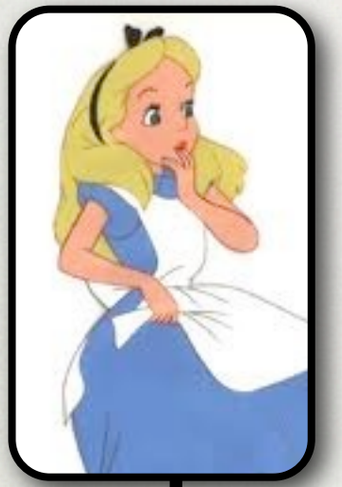
Proble

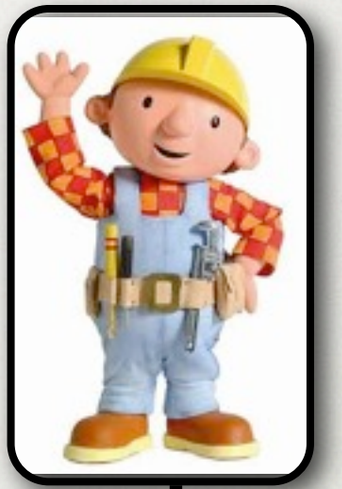
How can Alice and Bob *reliably* communicate over an *unreliable* channel?

- p
- Bob doesn't know which bits get corrupted.

Error-Correcting Codes

10011





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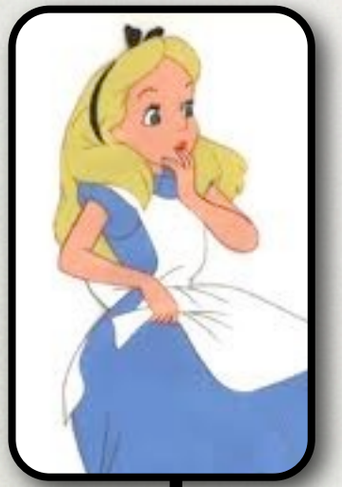

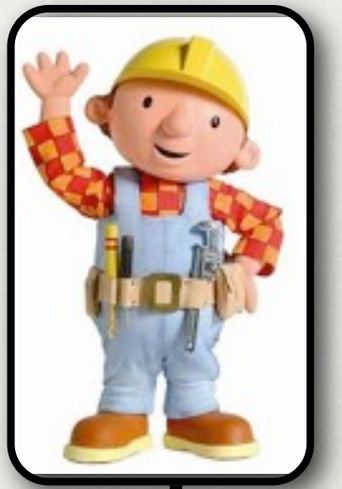

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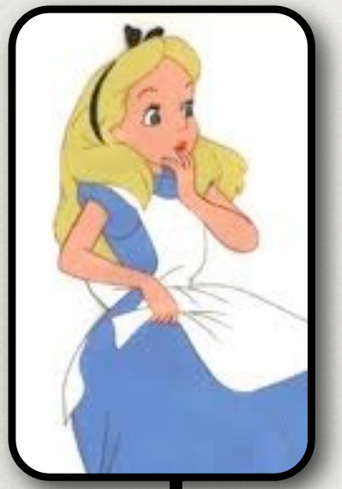

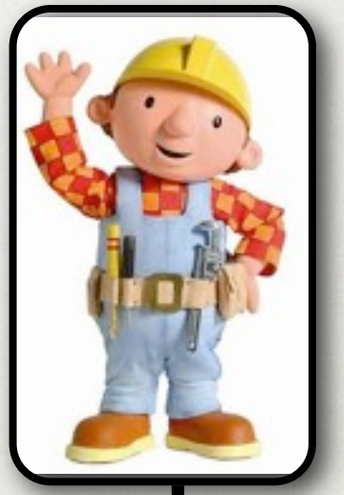

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Error-Correcting Codes

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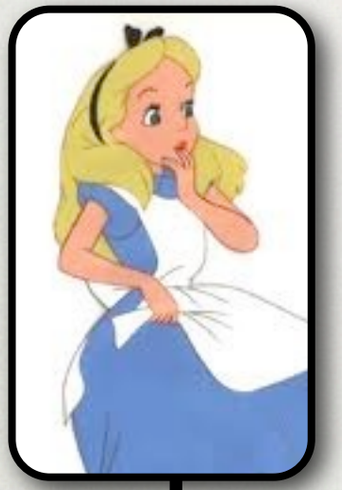

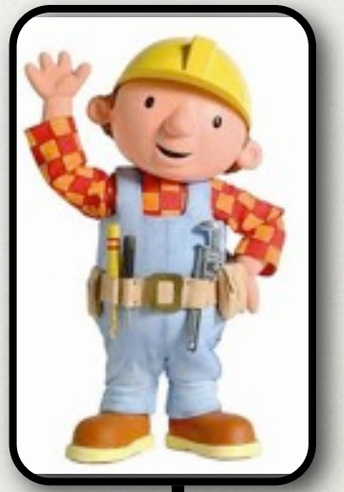

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Clicker Question

What is the rate and distance of a 5-repetition code?

(A) **(R, δ) = (1/3, 1/3)**

(B) **(R, δ) = (1/5, 1/5)**

(C) **(R, δ) = (1/5, 1)**

(D) **(R, δ) = (2/5, 1/5)**

(E) **(R, δ) = (1/5, 2/5)**

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Adjacency Matrix of Magic Graphs

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}

Adjacency Matrix of Magic Graphs

columns indexed by vertices in L

rows indexed by vertices in R

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
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Adjacency Matrix of Magic Graphs

$a_{ij} = 1$ *iff* (v_j, r_i) *is edge in magic graph*

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Magic Graph
Matrix Multiplication

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
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$*$

x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8

$=$

y_1
y_2
y_3
y_4
y_5
y_6

Magic Graph
Matrix Multiplication

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
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$*$

x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8

$=$

y_1
y_2
y_3
y_4
y_5
y_6

$$y_i = \sum_k a_{ik} * x_k$$

The Probabilistic Method