The Probabilistic Method

Week 10: Applications

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CS49/Math59

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Reading Quiz

What is a magic graph **G = (L ∪ R, E)**?

(A) a sparse bipartite graph

(B) a dense bipartite graph

(C) a bipartite graph where any v ∈ L has d neighbors

(D) a bipartite graph where any S ⊆ L has many neighbors

(E) multiple answers correct

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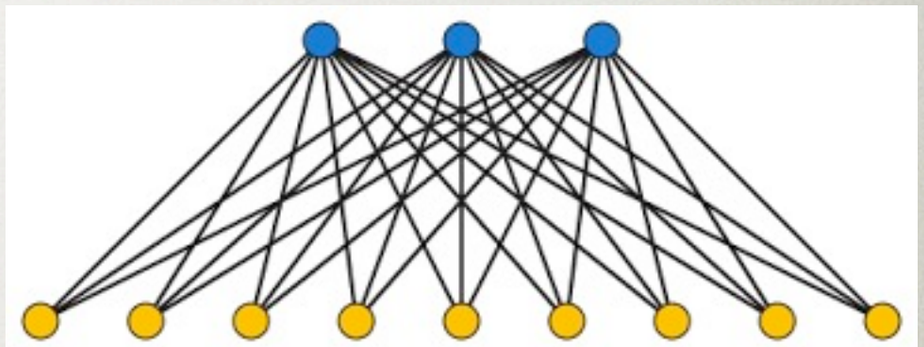
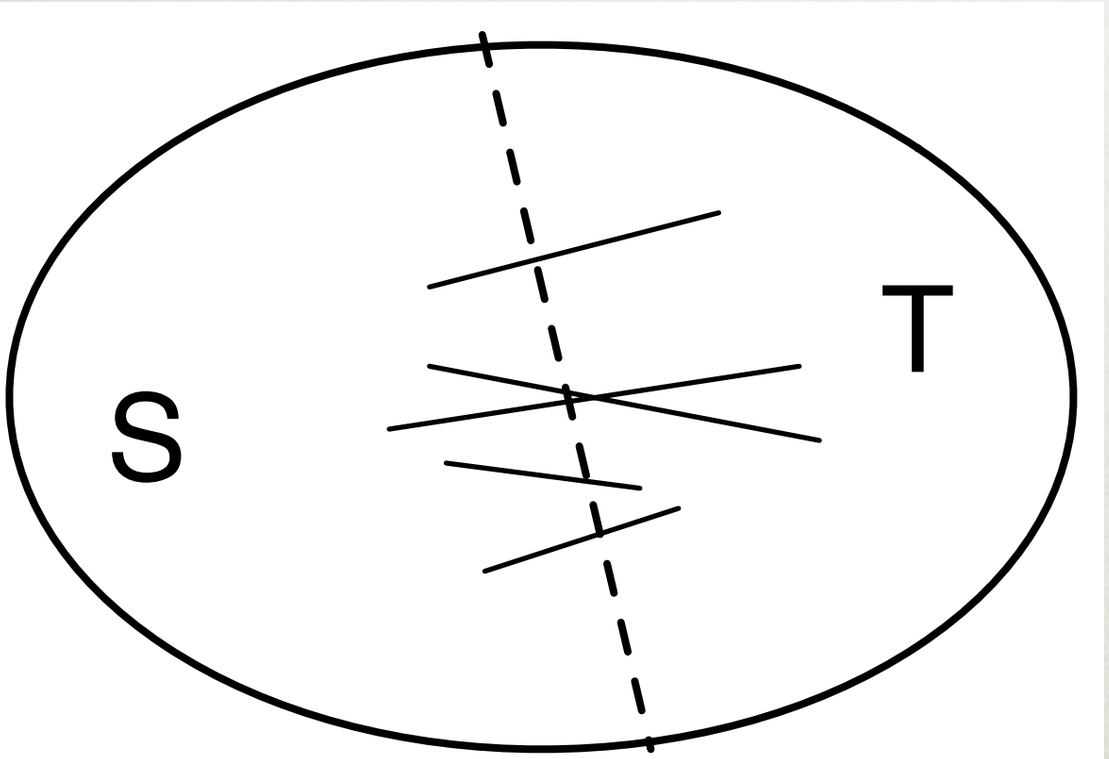
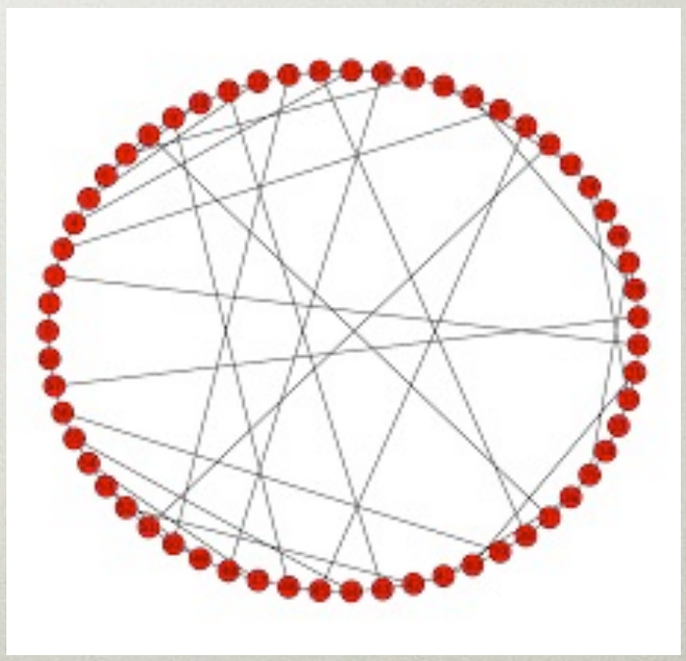
Expander Graphs

- *sparse* graphs with ***high connectivity***
- *d-regular*: each vertex has d neighbors
- graphs *"look random"*





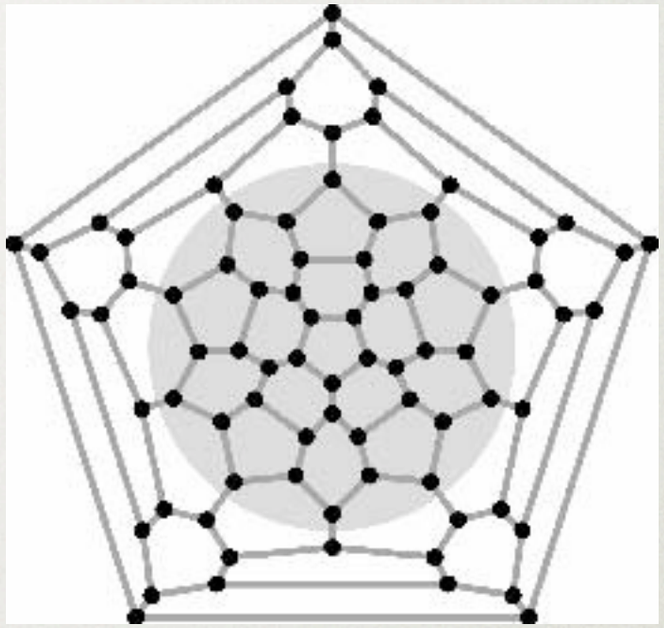
|E(S,T)| ≈ (d/n)*|S|*|T|

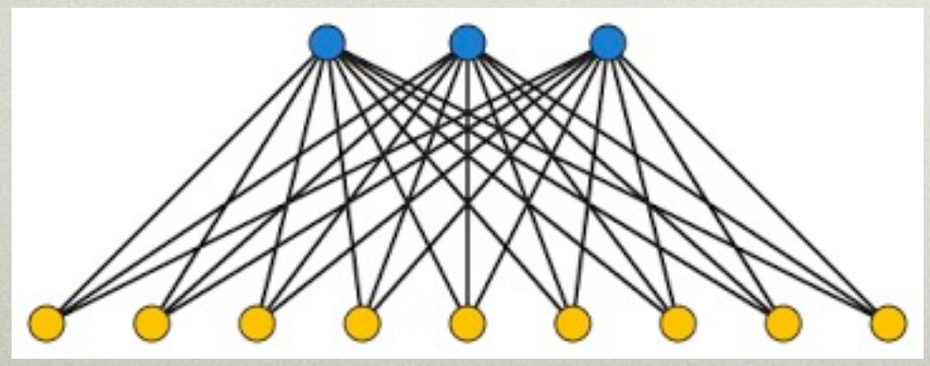


Why Expander Graphs?

Applications:

- derandomization
- coding theory
- error de-amplification
- cryptography
- complexity theory





More on Expanders:

- explicit constructions
- random walks
- spectral analysis

Randomized Algorithms

```
Algorithm A(x):
  count = 0;
  if heads {
    count++;
  } else {
    if rand(10)>5 {
      count += x;
      ...
    }
  ...
```

Randomized Algorithm A:

- input: x
- random string r
- output: YES, NO

One-sided error

- YES input: must output YES
- NO input: output NO w/prob > 15/16

Goal: minimize *runtime, space, amount of randomness,* ***error****, ...*

Clicker Question

What is $\Pr_R[A(x,R) \text{ error}]$?

(A) 1/3

(B) 1/4

(C) 1/10d

(D) 1/16

(E) none of the above

Clicker Question

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Clicker Question

How many times should you repeat **A(x,R)** to achieve error **< 1/d**?

(A) **O(2^d)**

(B) **O(d^2)**

(C) **O(d)**

(D) **O(log d)**

(E) **none of the above**

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Clicker Question

How many **v** ∈ **L** have all bad neighbors?

(A) at most d

(B) at most n/2

(C) at most n/10d

(D) at most d/n

(E) none of the above

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