The Probabilistic Method

Week 10: Applications

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CS49/Math59
Fall 2015

Reading Quiz

What is the Multiparty Pointer Jumping problem **MPJ**?

(A) **MPJ[i, f, x] := x[f(i)]**

(B) **MPJ[i, j, x] := x[i+j]**

(C) **MPJ[i, x] := xi**

(D) **MPJ[i, j, x] := x[i ⊕ j]**

(E) **None of the Above**

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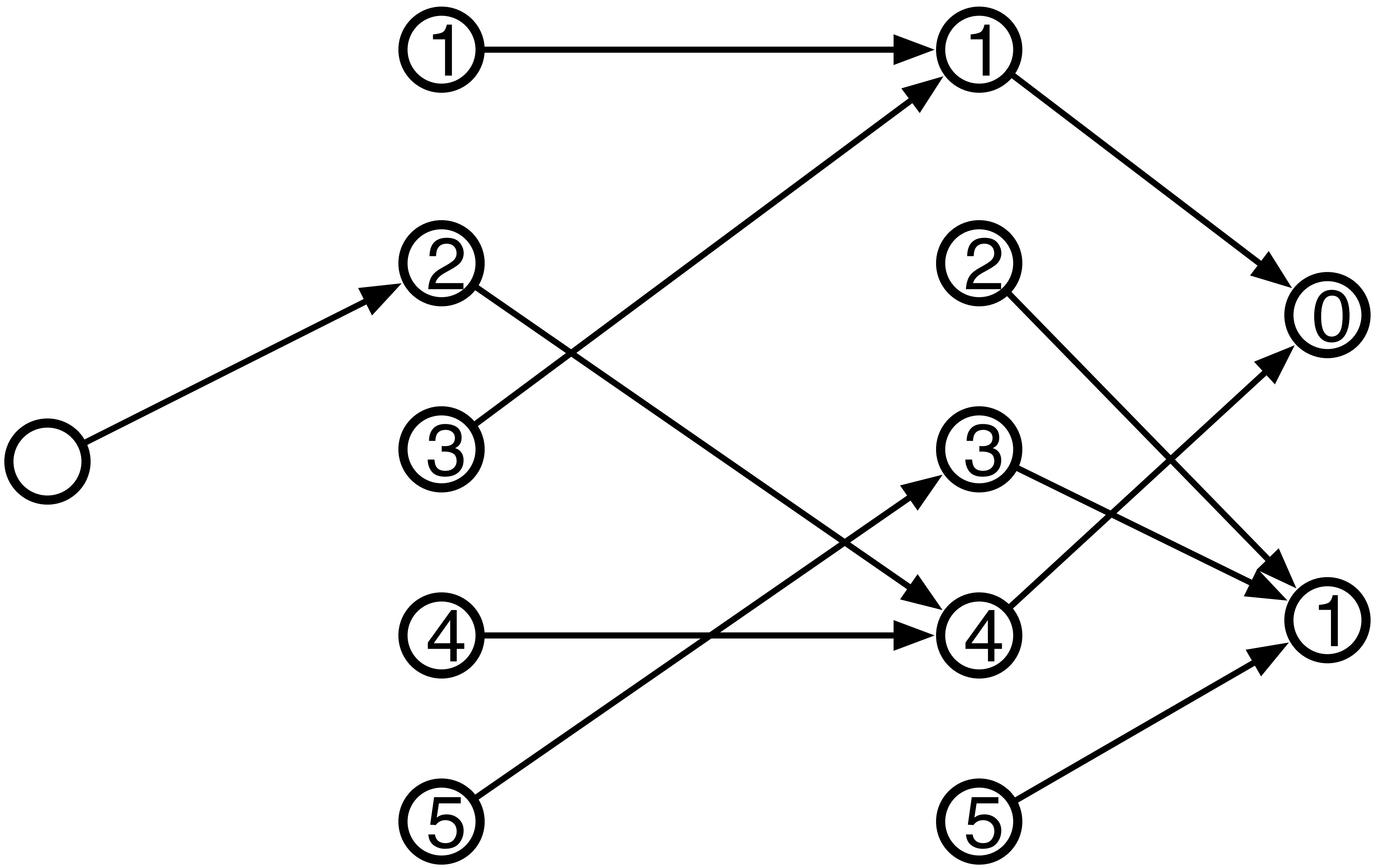
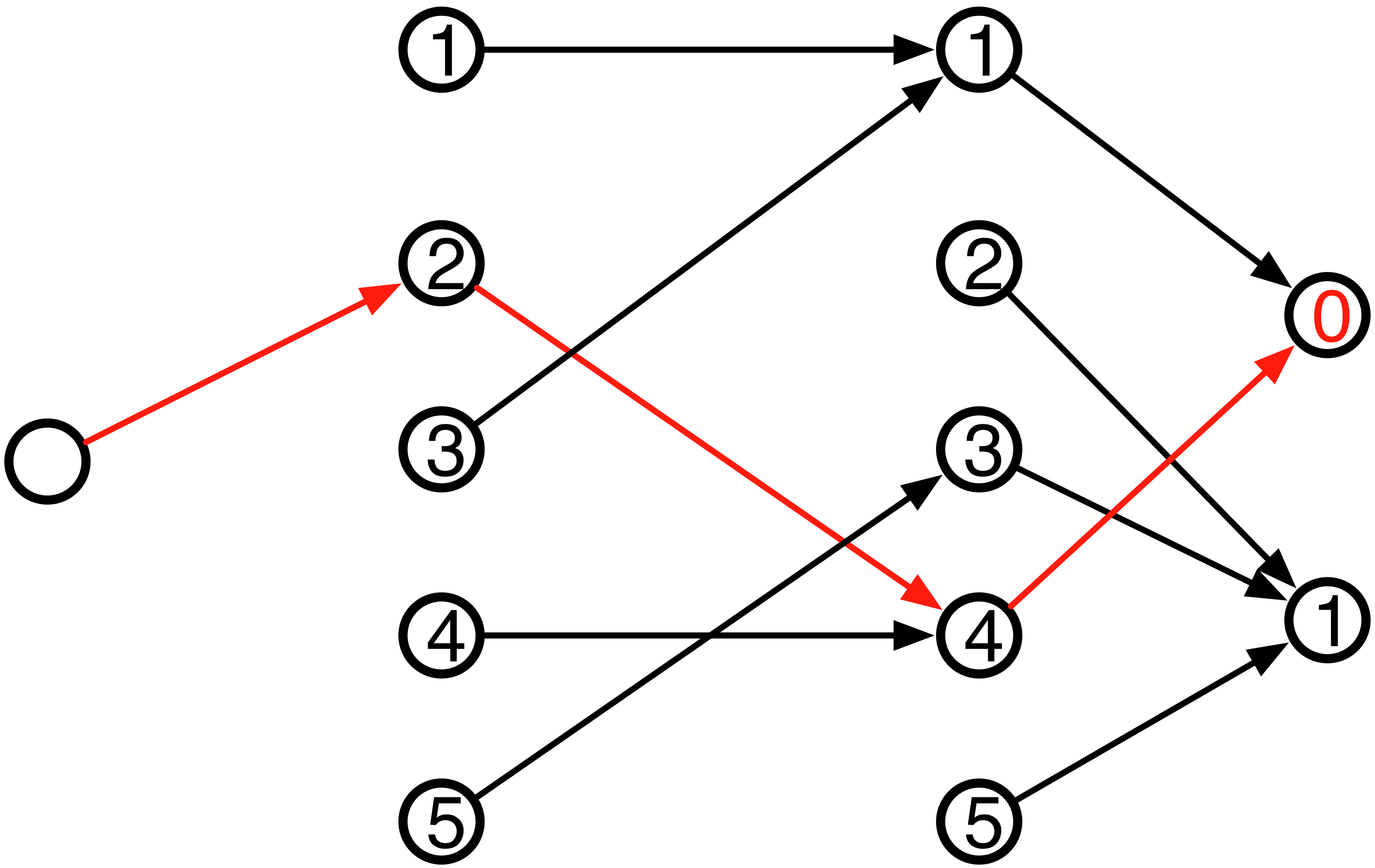
Multiparty Pointer Jumping



$i \in \{1,\ldots,n\}$ $f : \{0,1\}^n \rightarrow \{0,1\}^n$ $x \in \{0,1\}^n$

Output: $x[f(i)]$

Multiparty Pointer Jumping



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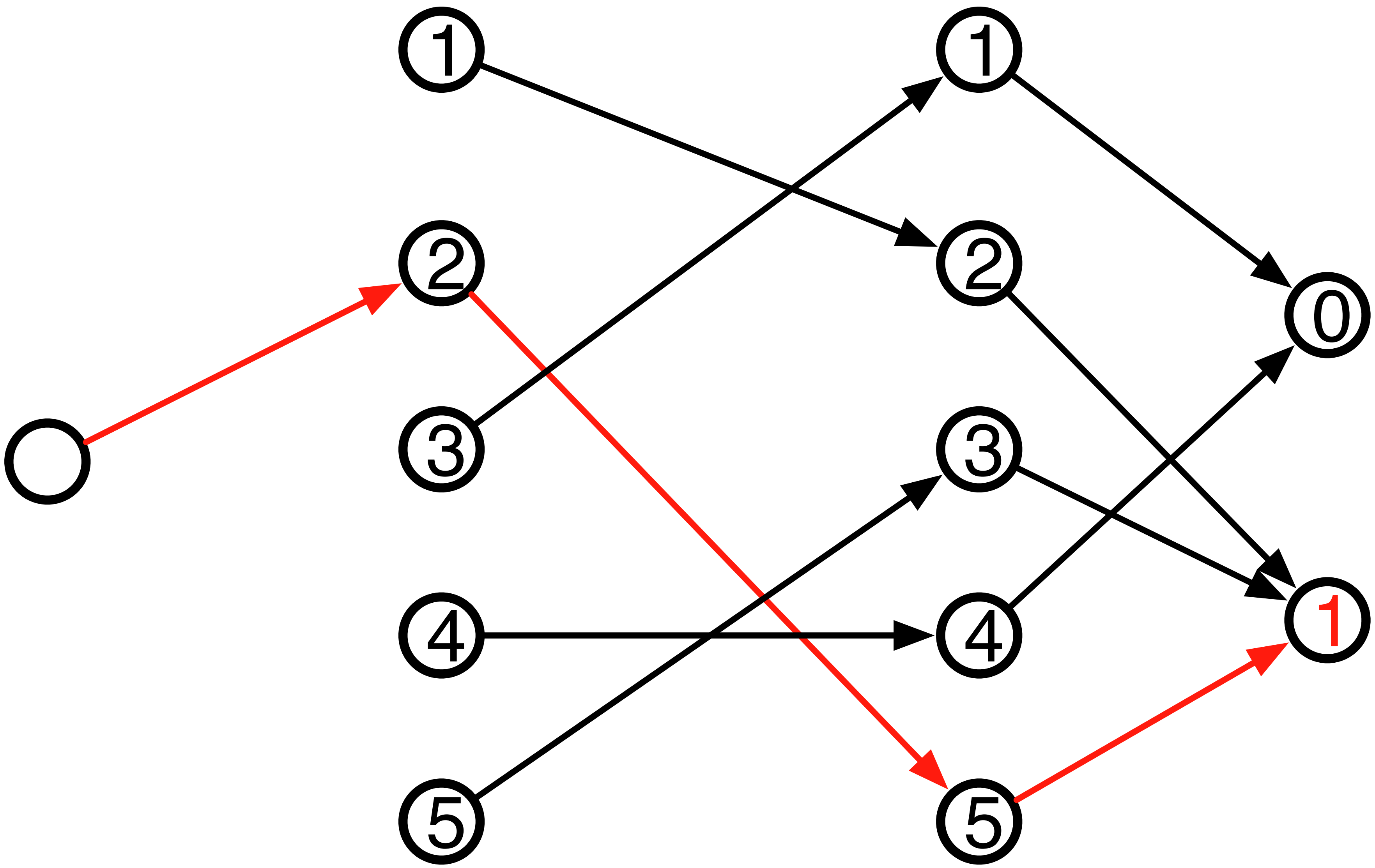
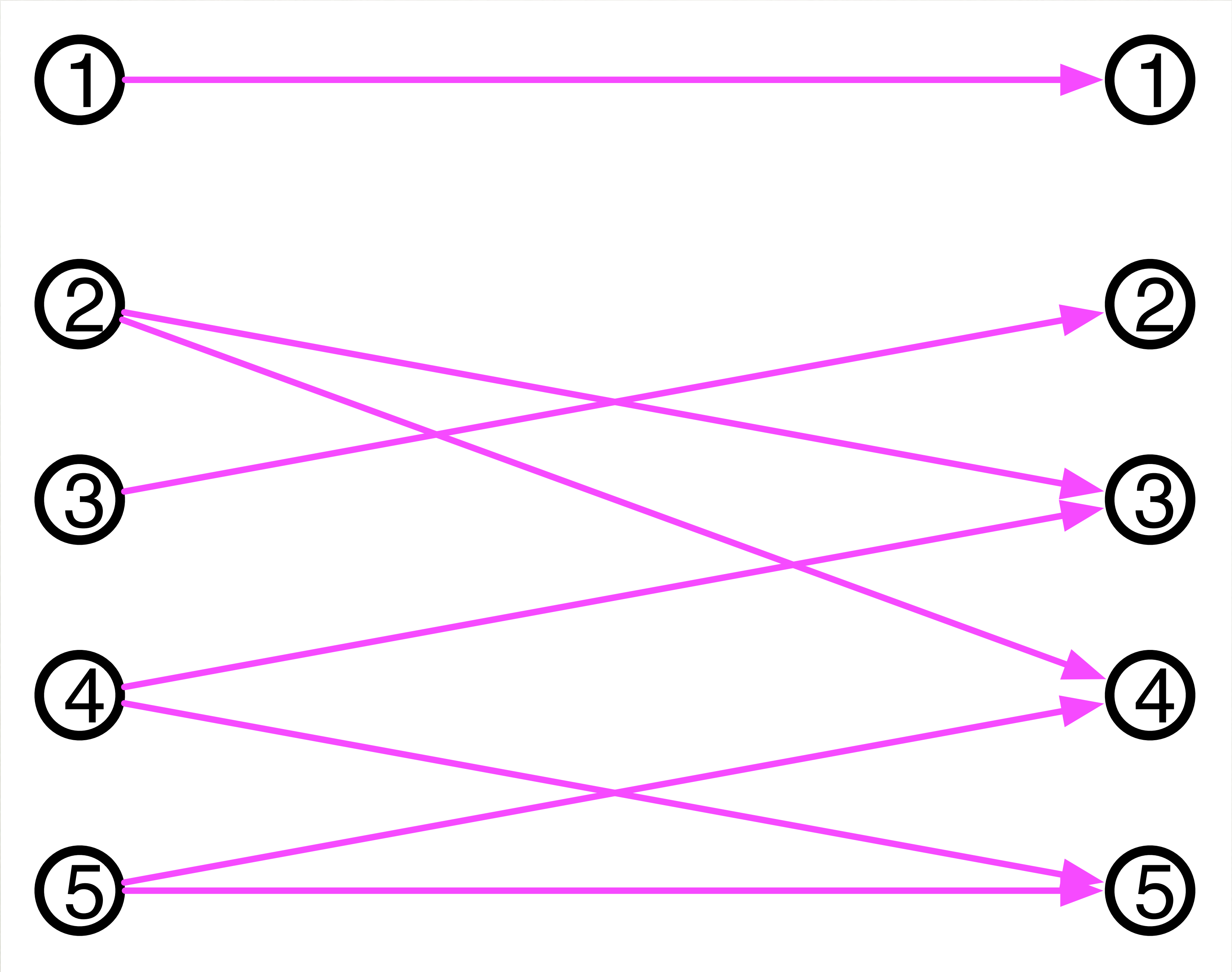
Permutation Pointer Jumping



$i \in \{1,\ldots,n\}$ $\quad$ $\pi \in S_n$ $\quad$ $x \in \{0,1\}^n$

Output: $x[\pi(i)]$

Protocol Ph: Creating $G_{\pi H}$



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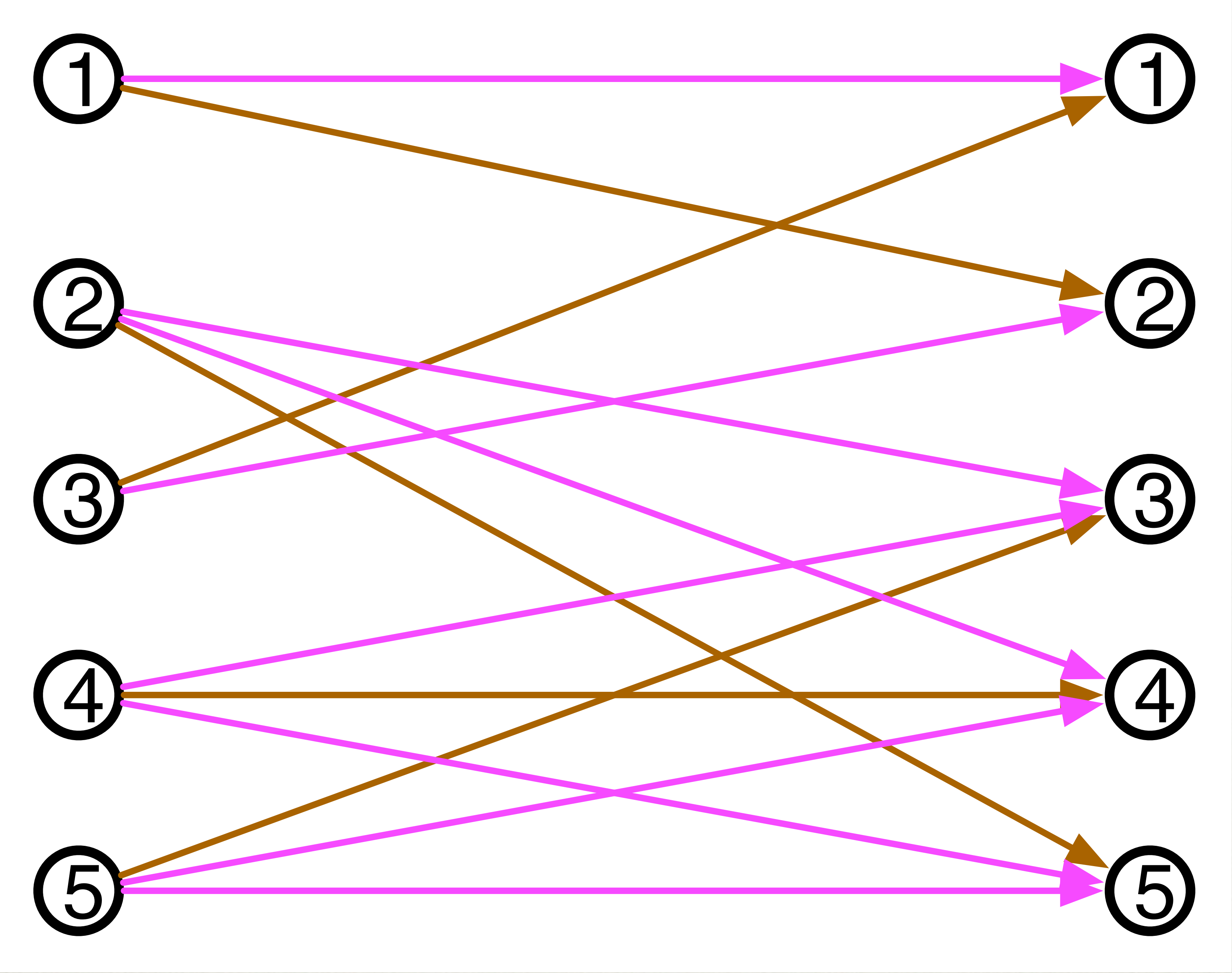


Clicker Question

Which edge is also in G_{π_H}?

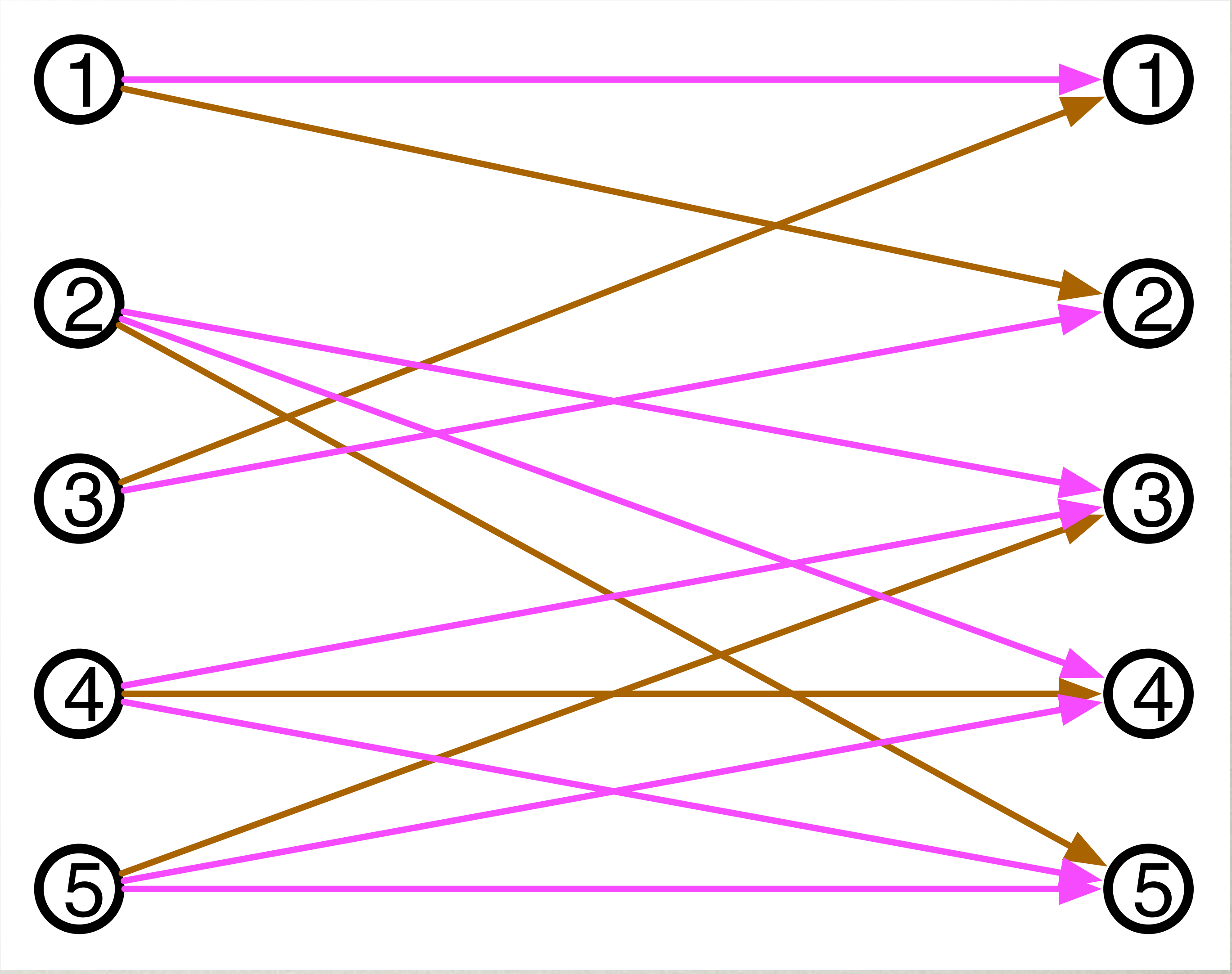
(A) (1,3)

(B) (2,3)

(C) (2,5)

(D) (4,5)

(E) None of the above



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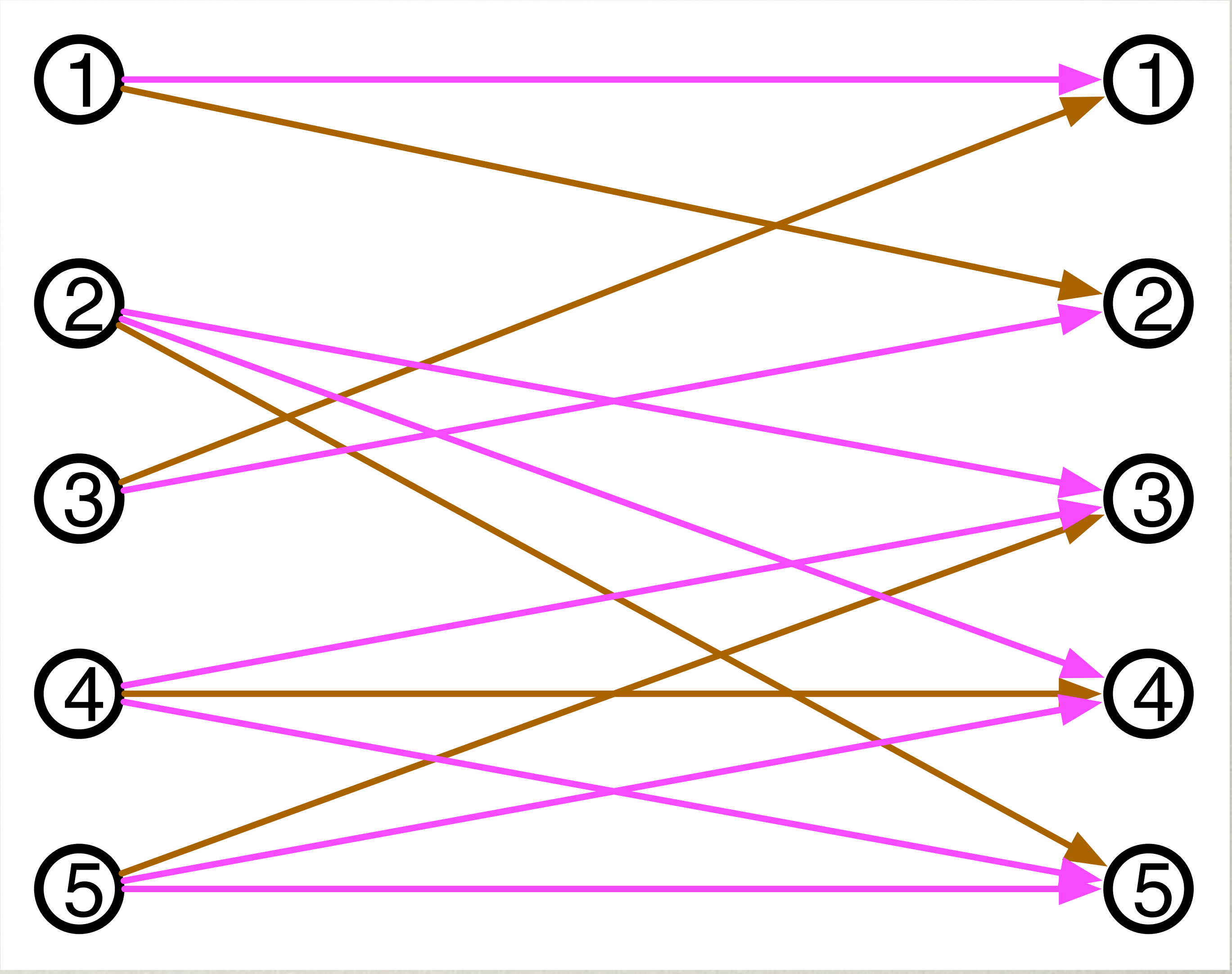

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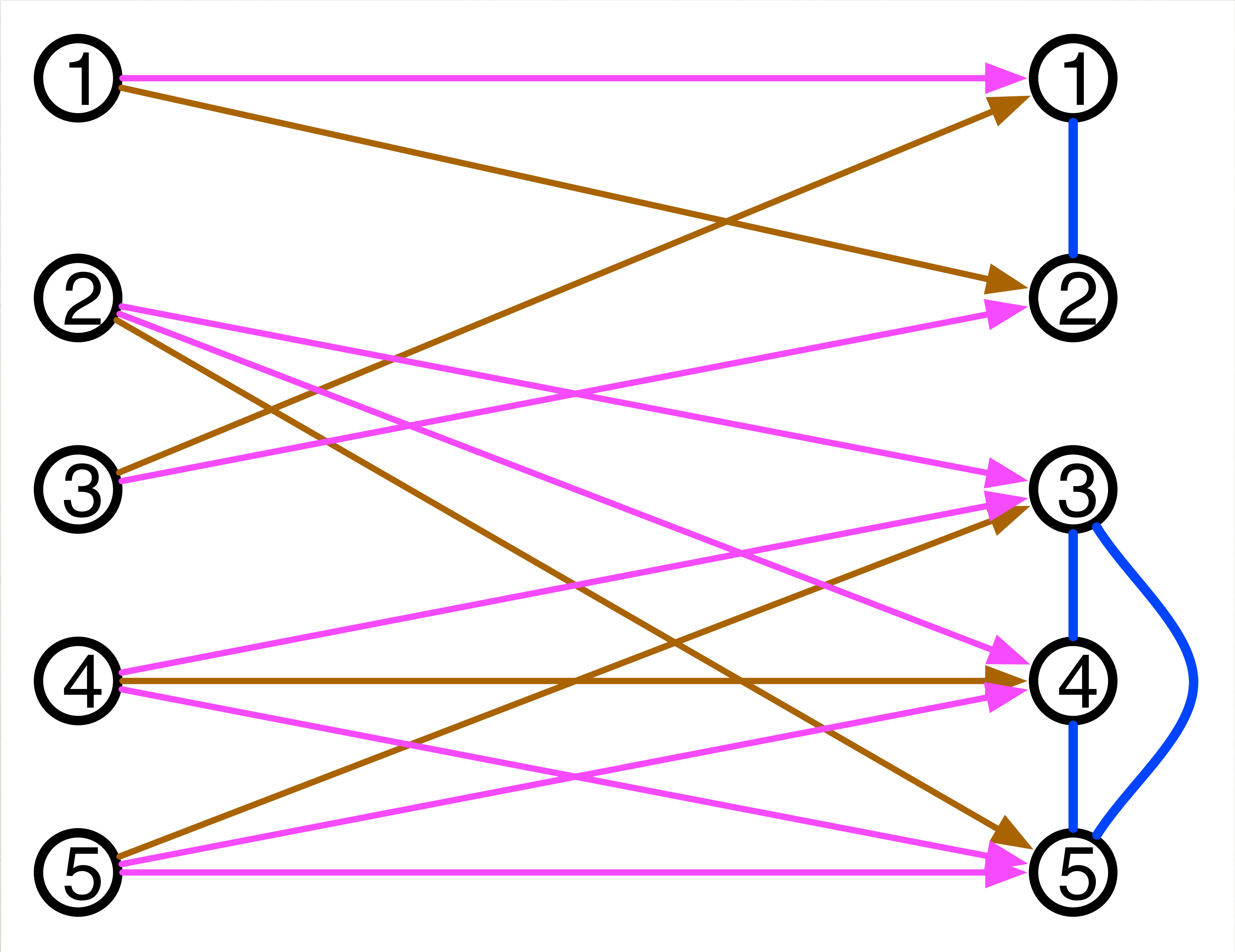

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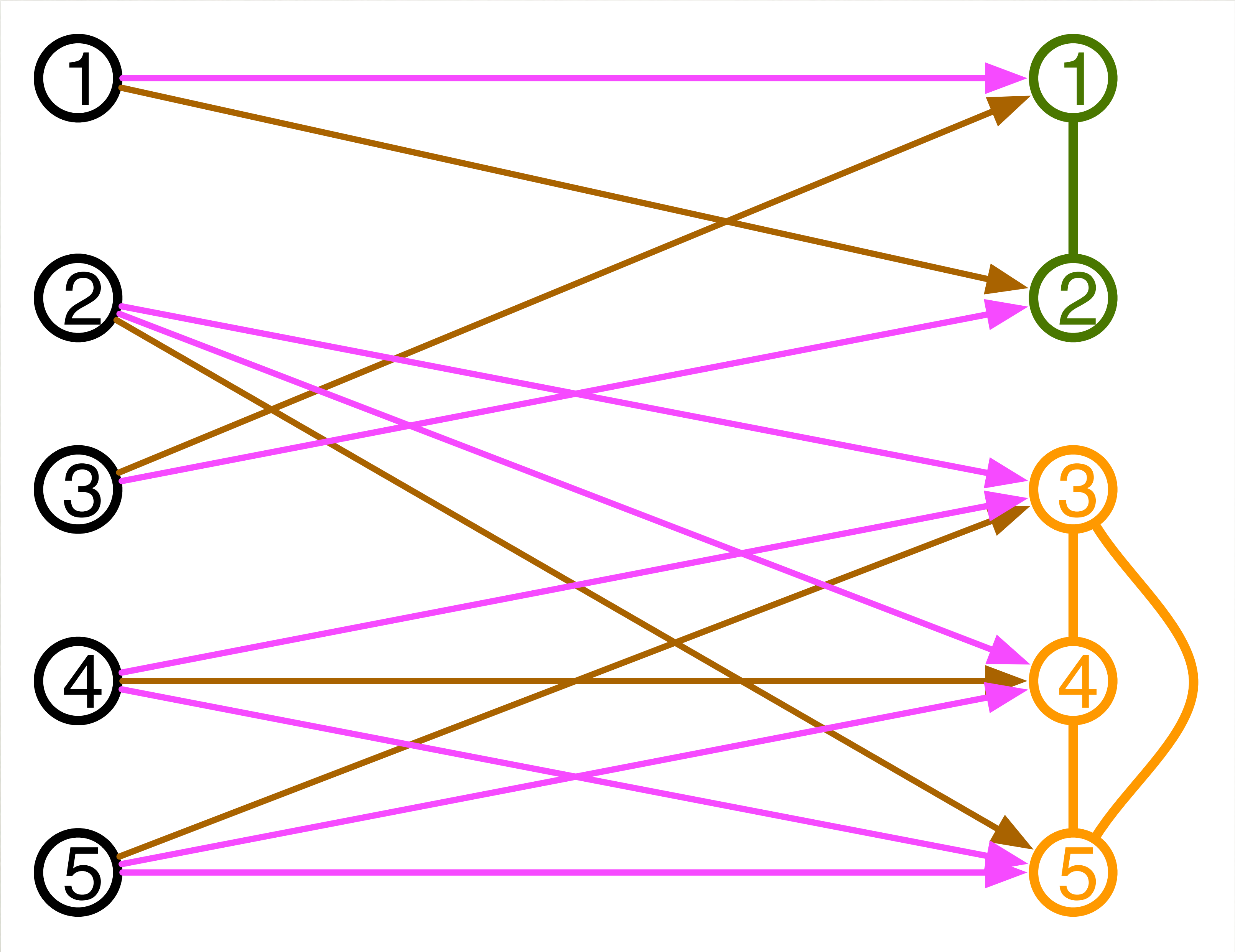

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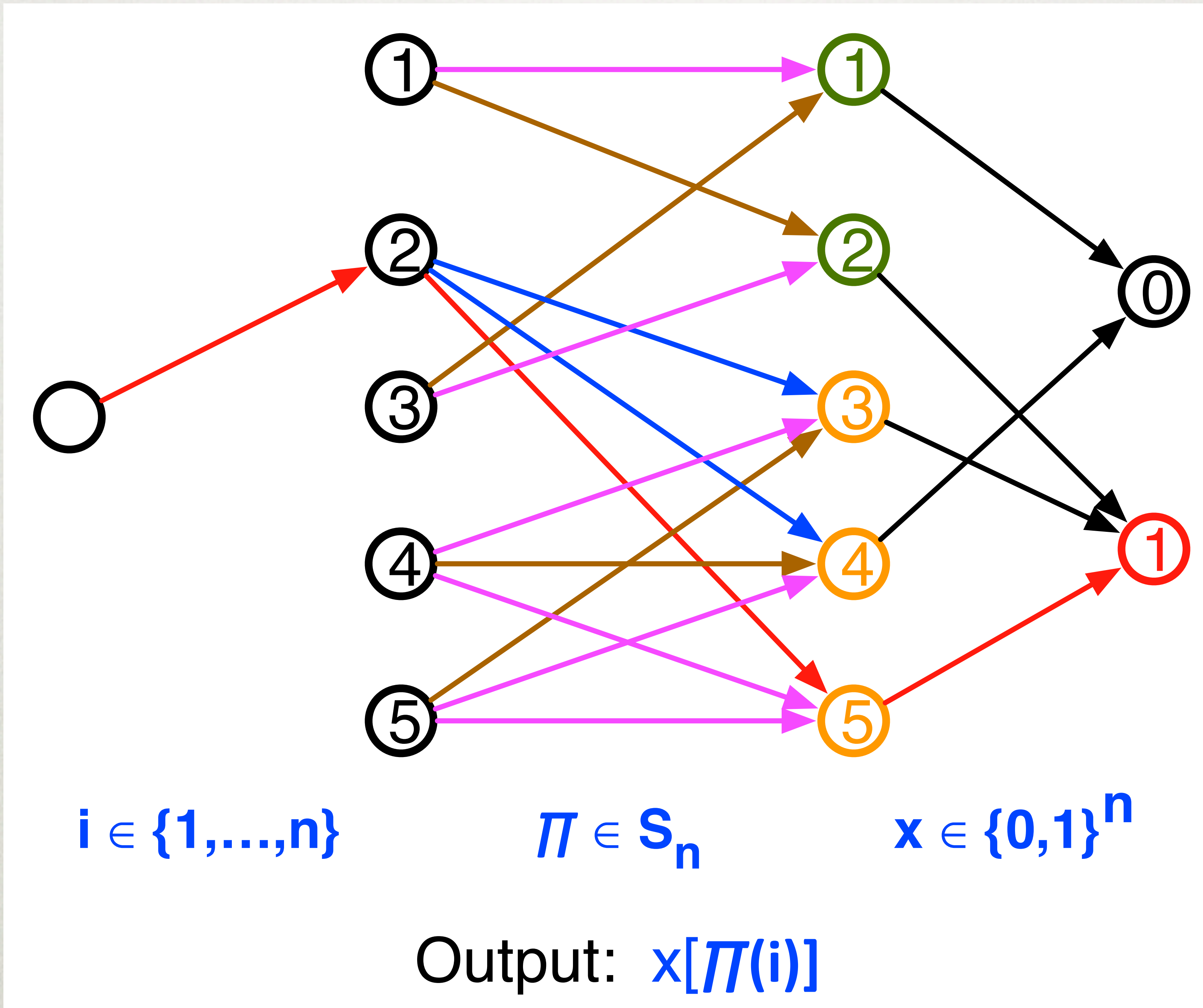


Clicker Question

Consider any vertex **i ∈ H**. What is **E[degree(i)]**?

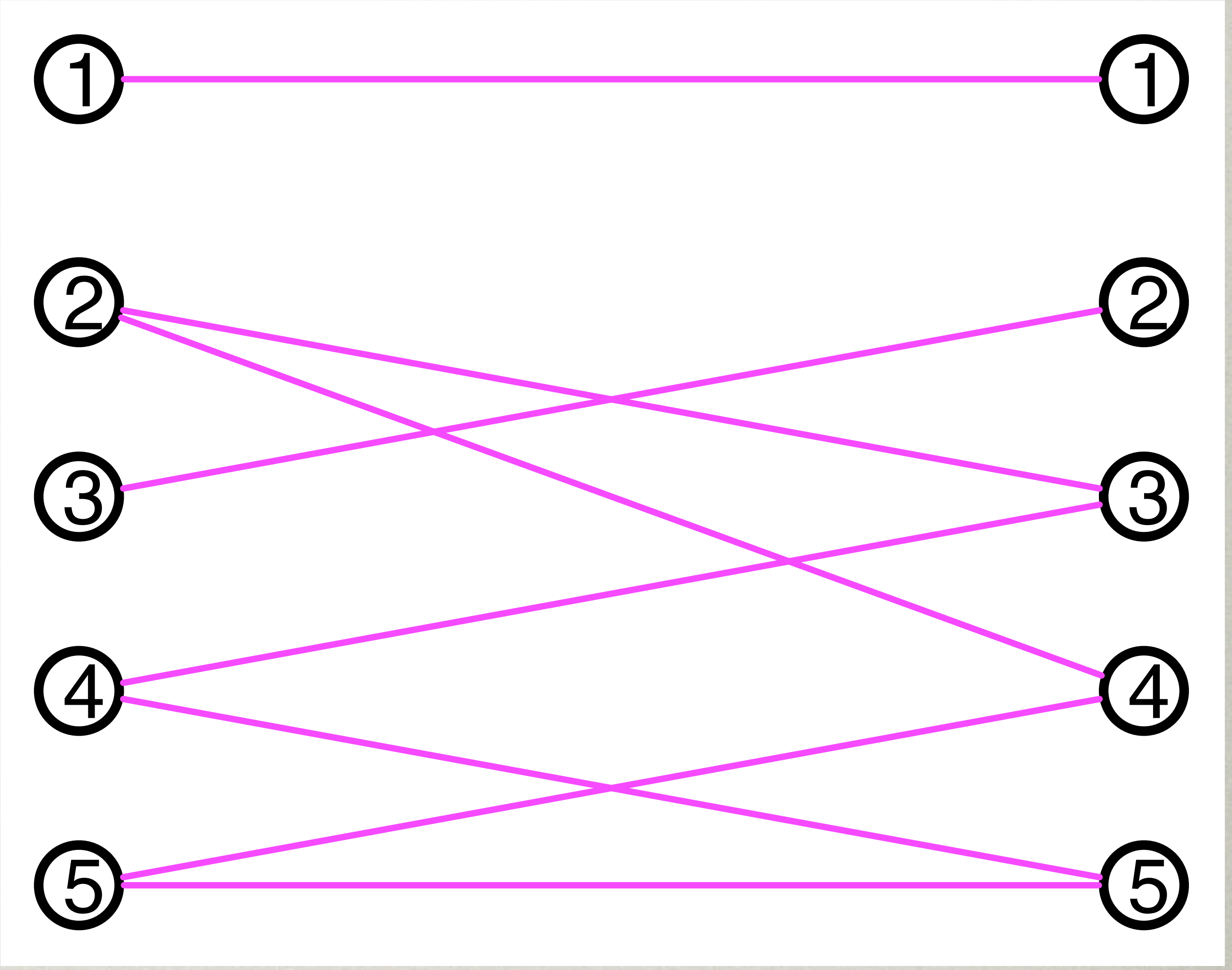
(A) **p**

(B) **pn**

(C) **(1-p)**

(D) **(1-p)n**

(E) **None of the above**



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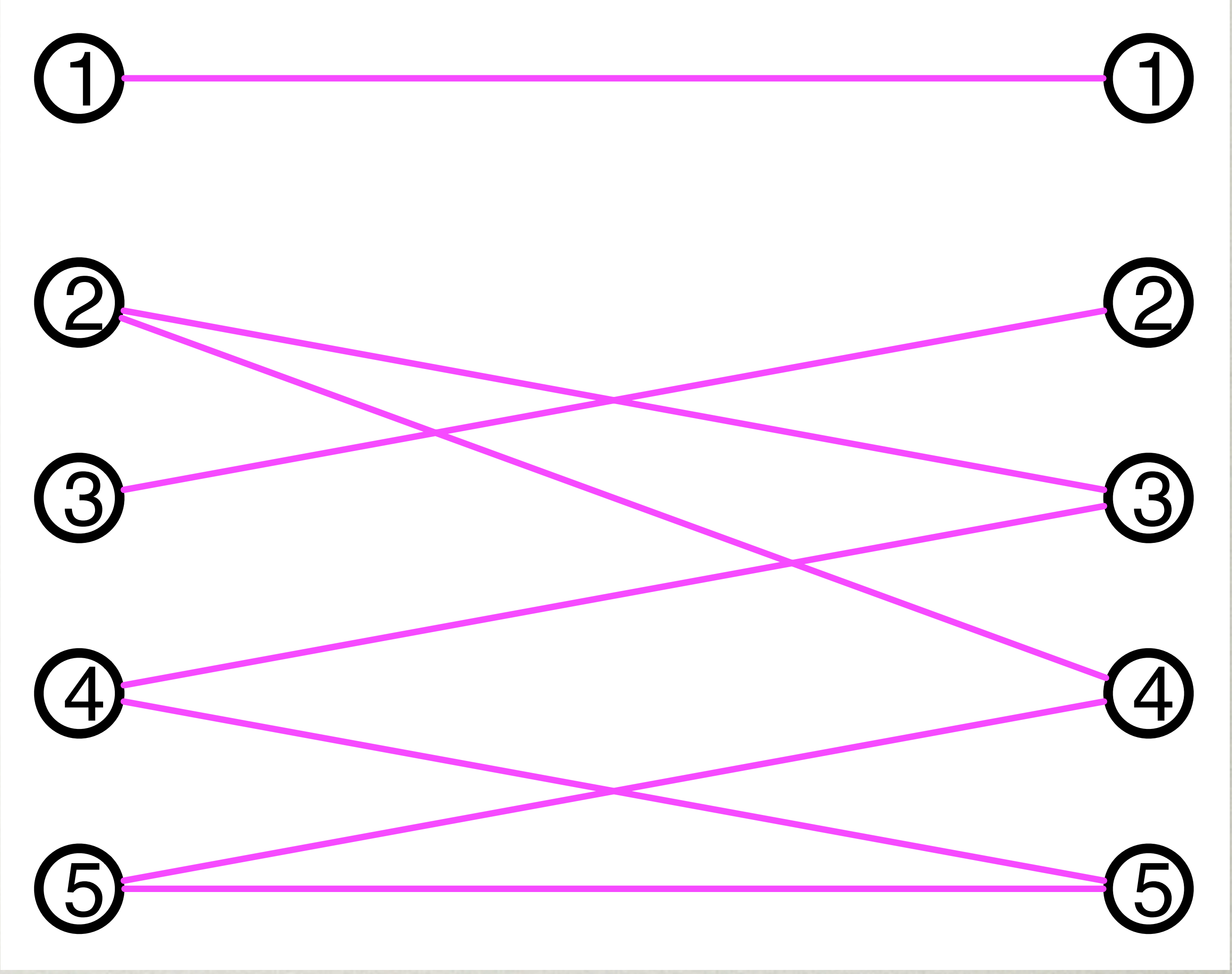
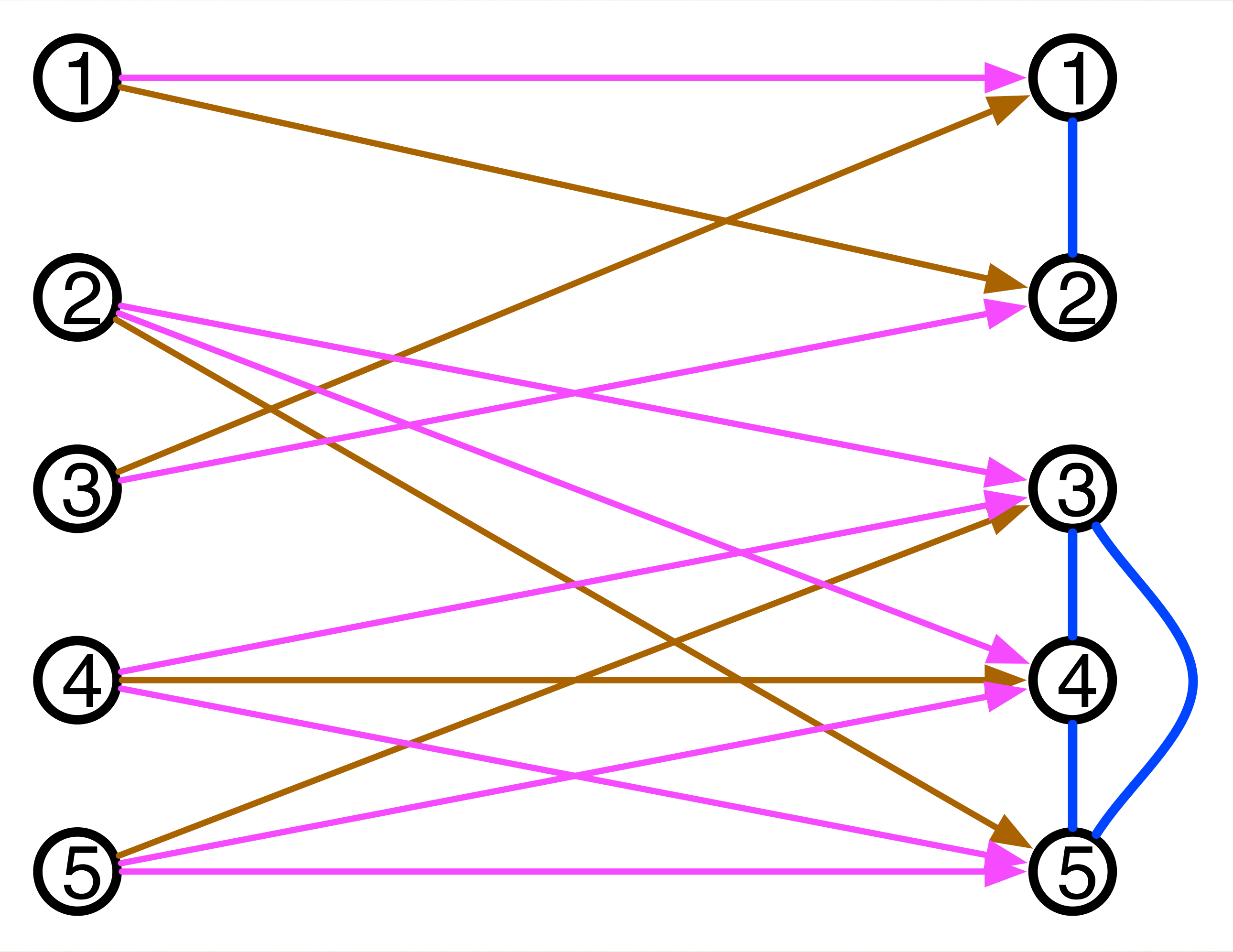


Clicker Question

Consider any possible edge (u,v) of G_{Π_H}.

What is **Pr[(u,v) ∈ G_{Π_H}]**?



(A) **p**

(B) **(1-p)**

(C) **p^2**

(D) **p(1-p)**

(E) **$1-p^2$**

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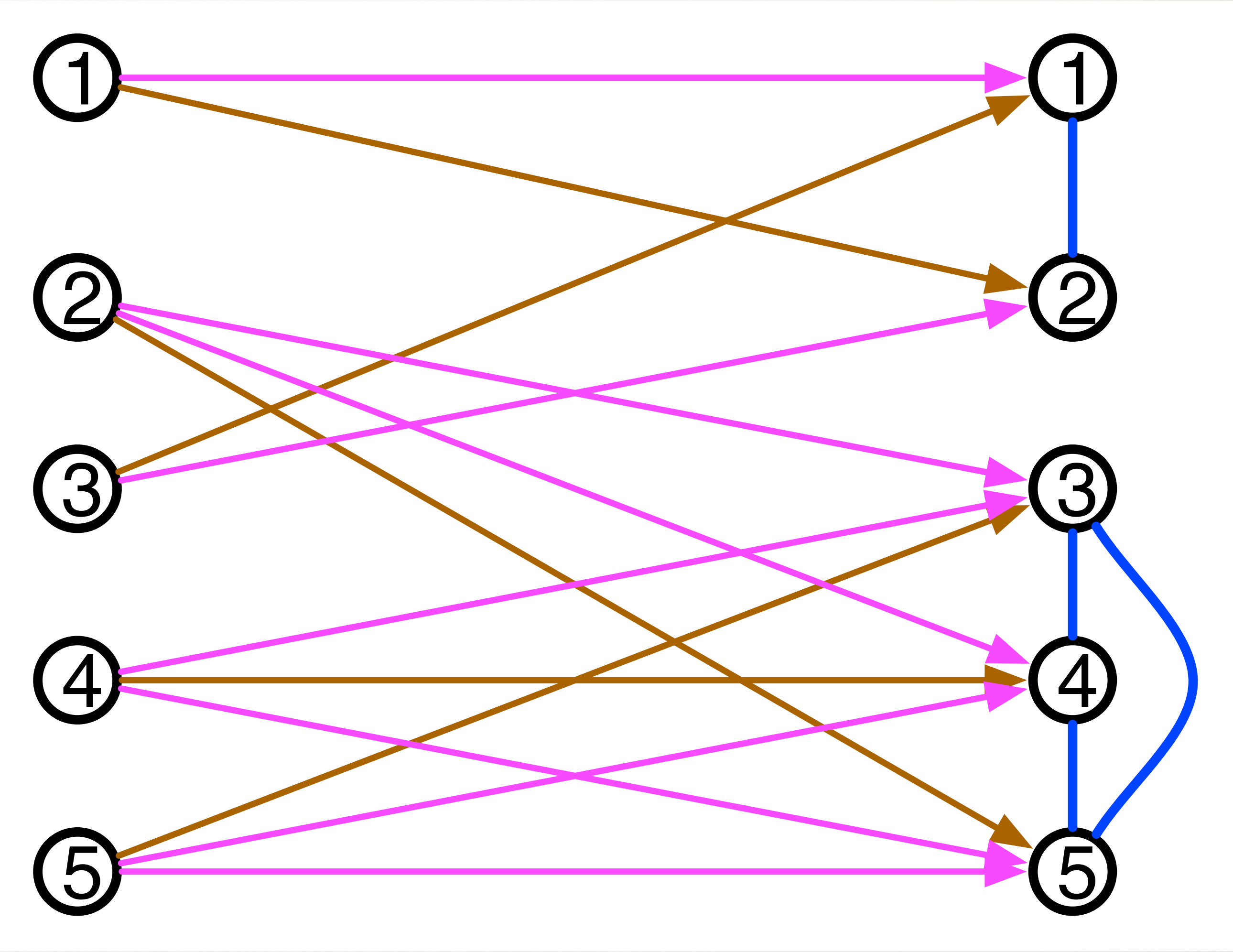

- **(A)** p
- **(B)** (1-p)
- **(C)** p^2
- **(D)** p(1-p)
- **(E)** $1-p^2$

Final Analysis

Claim: There exists **H** such that

$$\text{cost}(P_H) = O(n\log\log(n)/\log(n)$$

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Proof: consider random **H**, each edge w/prob **p**

- BAD_i : event that **i** has **> 2pn** neighbors
- BAD_{Π} : event that $G_{\Pi H}$ has **> 2nlog(1/p)/log(n)** cliques
- $BAD := \cup BAD_i \cup BAD_{\Pi}$
- Choose **p := loglog(n)/log(n)**
- $\Pr[BAD] \le n \Pr[BAD_i] + n! \Pr[BAD_{\Pi}] << 1$

Generalizing to MPJ

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PRS protocol works only for permutations.

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[Brody-Sanchez15]:

- adapts PRS for general functions
- edges in $\mathbf{G}_{\Pi H}$ no longer independent.
- ***Dependent Random Graphs***: each edge can depend on a few other edges.
- lower bound on **clique #**, upper bound on **chromatic number** still (asymptotically) the same.

The Probabilistic Method