CS49/Math59 Lab 4

This lab assignment is due **before the start of class** on Wednesday, October 21. Homework handed in during class but after I begin the lecture will be counted as late submissions. Some things to note:

- This is a **one week lab**. It is due the Wednesday after break. While you are welcome to work over break, you shouldn't need to, **assuming you start early.**
- I encourage you to write your solution using LaTeX, but you are not required to.
- You may have one partner for this assignment, but are not required to. If you work with a partner, submit just one writeup.
- Aside from your partner, you should not discuss problems in detail with anyone. It's OK to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else.

Make sure your names are on your submission, and show your work to maximize partial credit.

1. **Variance.**

   (Shoup 8.24) Let $X \in \{0, 1\}$ be a random variable. Show that $Var[X] \leq 1/4$.

2. **(Alon/Spencer 1.5)** Let $G = (V, E)$ be a graph on $n \geq 10$ vertices, and suppose that if we add to G any edge not in G, the number of cliques on ten vertices increases. Show that the number of edges in G is at least $8n - 36$.

   Hint: Use the Set Systems problem from lab. Let the elements that compose sets come from $\Omega := V$. For each non-edge (i, j) create a pair of sets (A_i, B_i). You'll need to choose A_i, B_i cleverly, so that for any i A_i, B_i are disjoint, but for $i \neq j$, the sets A_i, B_j intersect. Then appeal to the set systems theorem to show that there cannot be too many non-edges.

3. **(Alon/Spencer 2.9)** Let $G = (V, E)$ be a bipartite graph with n vertices, and for each $v \in V$, let $S(v)$ be a list of **more than** $\log_2 n$ colors. Prove there is a proper coloring[1] of V such that each vertex v is colored some color from $S(v)$.

   Hint: Let L, R be the two sides of the bipartite graph (i.e. $V = L \cup R$, and the only edges connect vertices in L to vertices in R). Try to cleverly assign the colors to L and R so that no color is used on vertices both in L and in R. As long as every vertex in L has some color in their list that's been assigned to L, argue that you're safe to use this color.

4. **Attribution.** Did you get assistance on any of the problems on this assignment from anyone aside from me and/or your lab partner? For example, did you discuss any problems at a high level with other students? Did you accidentally stumble on solutions while doing a websearch on related material? If so, descibe the nature of the assistance here. (e.g. "We briefly discussed problem 1 with X,Y, and Z" or "We saw a solution on ⟨this website⟩ before finding our own solution") If you (and your partner) worked alone, please say so here.

[1] a "proper coloring" of G is a coloring of the vertices so that no edge has endpoints colored the same.

5. **Lab Questionnaire.** (None of these questions will have an impact on your grade, this is to help provide the feedback I need to make the course the best it can be)

   (a) Approximately how many hours per partner did you spend on this lab?

   (b) How difficult did you find this lab? (enter a number 1-5, with 5 being very difficult and 1 being very easy)

   (c) Describe the biggest challenge you faced on this lab.