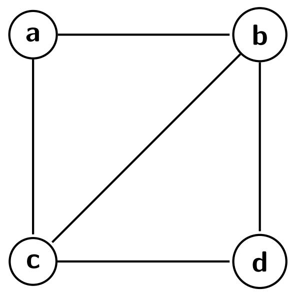
CS41 Lab 11: Traveling Salesman, Approximation Algorithms

Thursday, April 9

1. Consider the following graph $G = (V, E)$



with the following vertex weights: $w_a = 2, w_b = 3, w_c = 3, w_d = 2$.

(a) Give a minimal vertex cover.

(b) What is the minimum-weight vertex cover? what is the minimum weight?

(c) What is the optimal solution returned by the linear program we saw in class earlier this week? Note: don't worry about finding the absolute optimal answer. However, you should at least find a feasible solution that has less weight than the vertex cover you saw in the first part.

(d) What is the weight of the vertex cover returned by the LP-based approximation algorithm? How does it compare to the min-weight vertex cover?

2. **Traveling Salesman Problem.** In this problem, a salesman travels the country making sales pitches. The salesman must visit n cities and then return to her home city, all while doing so as cheaply as possible.

The input is a complete graph $G = (V, E)$ along with nonnegative edge costs $\{c_e : e \in E\}$. A *tour* is a simple cycle $(v_{j_1}, \ldots, v_{j_n}, v_{j_1})$ that visits every vertex exactly once.[1] The goal is to output the minimum-cost tour.

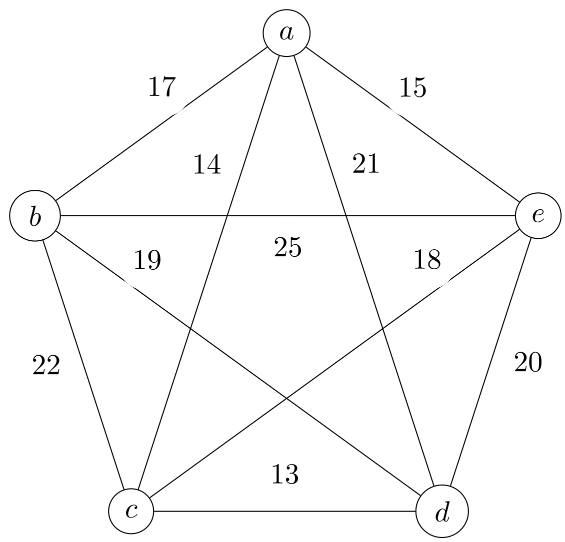
For many TSP applications (such as when the cost is proportional to the distance between two cities), it makes sense for the edges to obey the *triangle inequality*: for every i, j, k, we have

$$c_{(ik)} \leq c_{(ij)} + c_{(jk)}.$$

This version is often called METRIC-TSP.

The (decision version of the) Traveling Salesman Problem is NP-COMPLETE. For this problem, you will develop a 2-approximation algorithm for METRIC-TSP.

(a) First, to gain some intuition, consider the following graph:



(b) *On your own* try to identify a cheap tour of the graph.

(c) Build some more intuition by computing the minimum spanning tree (MST) of the graph. Let T be your minimum spanning tree.

(d) Let OPT be the cheapest tour. Show that its cost is bounded below by the cost of the MST: $\text{cost}(T) \leq \text{cost}(OPT)$.

(e) Give an algorithm which returns a tour A which costs at most twice the cost of the MST: $\text{cost}(A) \leq 2\,\text{cost}(T)$.

(f) Conclude that your algorithm is a 2-approximation for METRIC-TSP.

[1] except for the start vertex which we visit again to complete the cycle