CS41 Lab 12: NP-Completeness

November 30, 2020

In this lab, you will work on proving decision problems are NP-COMPLETE. Recall that to show a problem X is NP-COMPLETE, it suffices to:

1. Show $X \in \text{NP}$. (In otherwords, construct a polynomial-time verifier for L.)
2. Pick a problem Y known to be NP-COMPLETE.
3. Reduce $Y \leq_{\text{P}} X$.

   Given an instance S_Y of problem Y, show how to construct an instance S_X for problem X (in polynomial time!) such that

   (a) If S_Y is a YES input for problem Y then S_X is a YES input for problem X.

   (b) If S_Y is a NO input for problem Y then S_X is a NO input for problem X.

Show that the following problems are NP-COMPLETE.

1. 3-SAT. Given n boolean variables $x_1, \ldots, x_n$ and m clauses $c_1, \ldots, c_m$ such that each clause is the OR of three literals (e.g., $c_i = x_1 \vee \bar{x}_2 \vee x_4$), output YES iff there is a truth assignment to $x_1, \ldots, x_n$ that satisfies all clauses.
2. SUBSET-SUM. Given a list of n items with weights $w_1, \ldots, w_n$, along with a weight threshold W, output YES iff there exists a subset $S \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in S} w_i = W$.
3. INTERSECTION-INFERENCE. In this problem, the inputs are
   - a finite universe U of n items,
   - a collection $A_1, \ldots, A_m \subset U$ of subsets of U, and
   - a collection of m integers $c_1, \ldots, c_m$.

   Output YES iff there is a subset of the universe $X \subset U$ such that for every i, X intersects with A_i in exactly c_i elements. In other words, output YES iff there exists $X \subset U$ such that for all $1 \leq i \leq m$, we have $|X \cap A_i| = c_i$.