CS41 Homework 3

This homework is due at 11:59PM on Sunday, September 27. Write your solution using LaTeX. Submit this homework using **github** as a file called **hw3.tex**. This is an individual homework. It's ok to discuss approaches at a high level. In fact, we encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. The only exception to this rule is work you've done with a lab partner *while in lab*. In this case, note (in your **README.md file**) who you've worked with and what parts were solved during lab.

1. **Analysis.** Let $f(n) = 12n^{4/5}$ and $g(n) = n^{3/5}(\log n)^6$. Prove that $g(n) = O(f(n))$. You may use techniques and facts from class and the textbook; your proof should be formal and complete.

2. **Asymptotic Rates of Growth.** Arrange the following functions in ascending order of growth rate. That is, if g follows f in your list, then it should be the case that $f = O(g)$.

   - $f_1(n) = n^2$
   - $f_2(n) = 6\log(n) + 10$
   - $f_3(n) = n\log(n)$
   - $f_4(n) = 2 \cdot 2^n$
   - $f_5(n) = \sqrt{5n}$

   No proofs are necessary.

3. **The Skee-Ball Problem.** A group of six children were recently playing at the beach (socially distantly of course) when they found a bag full of 216 arcade game tokens. All six children LOVE Skee Ball, no naturally they are excited to get as many tokens as they can!

   Now, the six children would like to divide their new-found arcade wealth. Being the precocious childeren they are, they have decided to vote on how to best divide the tokens. The children's voting process is as follows. First, the oldest child proposes a scheme for dividing the tokens. For example, she might propose to keep 146 tokens for herself and give the five remaining children 14 tokens each. The children are democratic and precocious, but also a bit testy about proposals. If the majority vote against this scheme, then the rest of the children will make the oldest child go home without any tokens! In this case, the next oldest child will make a proposal, risking being sent home of more than half of the remaining children vote against him.

   The process repeats in a similar fashion (oldest remaining child proposes a way to divide the tokens, children vote, and the oldest child gets goes home if the proposal is rejected) until a division is accepted.

   How should the oldest child divide the bag of arcade game tokens? You can assume that all six children are greedy and care only about how many tokens they receive.

4. **(Extra Challenge)** Three people enter a room and have a each have Garnet or Gray hat placed on their heads. They cannot see their own hats, but can see the other hats.

   The color of each hat is purely random (i.e. w/probability 50% the hat is Garnet, w/probability 50% the hat is Gray).

   Each person needs to guess the color of their own hat by writing it on a piece of paper. They can also write "pass".

   They cannot communicate with each other in any way once the game starts but can discuss strategies before the game starts.

   The players each win ONE MILLION DOLLARS if at least one player guesses their hat color correctly **and** no player guesses their hat color incorrectly. If all players pass, or if any player guesses incorrectly, players lose the game and win nothing.

   Give a strategy that causes the players to win ONE MILLION DOLLARS with probabilty greater than 50%. Describe your strategy in words and rigorously analyze their winning probability.