CS41 Homework 11

This homework is due at 11:59PM on Friday, December 4. This is a six point homework. Write your solution using \LaTeX. Submit this homework using github as a .tex file. This is a partnered homework. You should primarily be discussing problems with your homework partner.

It’s ok to discuss approaches at a high level with others. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. The only exception to this rule is work you’ve done with a lab partner while in lab. In this case, note (in your homework poll file) who you’ve worked with and what parts were solved during lab.

1. **INTERSECTION-INFERENCE** (K&T 8.16) Consider the problem of reasoning about the identity of a set from the size of its intersections with other sets. You are given a finite set $U$ of size $n$, and a collection $A_1, \ldots, A_m \subseteq U$ of subsets of $U$. You are also given integers $c_1, \ldots, c_m$. The question is: does there exist $X \subseteq U$ such that for each $i = 1, 2, \ldots, m$, the cardinality of $X \cap A_i$ equals $c_i$. We will call this an instance of the INTERSECTION-INFERENCES problem, with input $U, \{A_i\}, \{c_i\}$.

Prove that **INTERSECTION-INFERENCE** is NP-complete. **Hint:** reduce from the following problem, which you may assume is NP-complete:

**Problem One-In-Three-Sat:**

**Inputs:** $n$ variables $x_1, \ldots, x_n$ and $m$ clauses $c_1, \ldots, c_m$ where each clause is the OR of three literals e.g., $c_i = (x_1 \lor \overline{x_2} \lor x_3)$.

**Output:** yes iff there is a truth assignment to the variables such that for each clause there is exactly one satisfied variable.

**Hint:** Let $U$ be the set of literals. You’ll have to work to ensure that a variable and its negation cannot both end up in $X$.

2. **PATH-SELECTION** (K&T 8.9) Consider the following problem. You are managing a communication network, modeled by a directed graph $G = (V, E)$. There are $c$ users who are interested in making use of this network. User $i$ (for each $i = 1, \ldots, c$) issues a request to reserve a specific path $P_i$ in $G$ on which to transmit data.

You are interested in accepting as many of these path requests as possible, subject to the following restriction: if you accept both $P_i$ and $P_j$, then $P_i$ and $P_j$ cannot share any nodes.

Thus, the PATH-SELECTION problem asks: Given a directed graph $G = (V, E)$, set of requests $P_1, \ldots, P_c$—each of which is a path in $G$, and a number $k$, output yes iff it is possible to select at least $k$ paths so that no two of the selected paths share any nodes.

Prove that **PATH-SELECTION** is NP-complete.

3. **(Extra Challenge Problem.)** Show that **One-In-Three-Sat** is NP-complete.

4. **(Extra Challenge Problem.)** Does $P = \text{NP}$? Answer yes or no. Justify your response with a formal proof.