CS41 Lab 6: Recurrence Relations

The lab problems this week center around recurrence relations. Recurrence relations are important tools for modeling and analyzing the efficiency of algorithms. The purpose of this lab is to gain practice using the different techniques for solving recurrence relations.

The following equalities might be helpful:

- For any $a, b > 0$, $\log(a^b) = b \log a$
- For any $a, b \geq 1$, $a^{\log b} = b^{\log a}$.
- For any $a, b \geq 1$, $\log(a/b) = \log(a) - \log(b)$.
- For any $x \in \mathbb{R}$, $2^{\log x} = x$

Try to use both the Recursion Tree Method and the Substitution Method to solve these recurrence relations. You do not need to solve these exactly, only asymptotically. e.g. if a recurrence relation has an exact solution of $T(n) = 3n^2 - 19\sqrt{n}$, then an answer of $T(n) = O(n^2)$ suffices.

1. The following summation appears often in recurrence relations. Fix any constant $c > 0$. Using induction, show that for all $m > 0$, we have

$$\sum_{k=0}^{m-1} c^k = \frac{c^m - 1}{c - 1} \ .$$

   Note: Feel free to skip this problem if you feel comfortable with induction.

2. $S(n) = S(n-1) + 3n$,
   $S(1) = 3$

3. $M(n) = 3M(n/2) + 2n$,
   $M(1) = 1$

4. $W(n) = 3W(n/3) + n^2$,
   $W(1) = 1$

5. $H(n) = 4H(n/2) + 2n^2$,
   $H(4) = 5$

6. $T(n) = 3T(n/3) + 10\sqrt{n}$,
   $T(1) = 5$.