Lab 3: Algorithm Analysis

Due: Sunday, September 28 at 11:59PM

Overview

In this lab, you will answer several math and algorithm questions and complete the mystery function exercise. You will submit a hard copy of this lab. You may type your solution, if you wish. **This is an individual lab**. You can retrieve requisite code from `update35` for the mystery function portion of the lab.

Deliverables

Your submission should be **concise and easy to read**. You should answer all 4 short answer questions, the run times of all 6 functions, written support of the mapping to mystery functions, and at-least two print out of graphs to support that argument.

Submission

You will submit this lab in hard copy to my mail slot outside my office, Science 260.

Short Answer Questions

1. Justify the following Big-O for the given functions:
   (a) $2n^3 - 0.5n^2 + 5n + 10$ is $O(n^3)$
   (b) $\frac{1}{2}n + 2log(n)$ is $O(n)$

2. Prove the following claim by induction: For all $n \geq 1$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

3. The function `checkMax`, given below, takes as input an array A of size n, and a number x, and should return true if A has an element bigger than x. Using loop invariants and induction, prove that `check Max` works correctly. HINT: formally, you are trying to prove S: checkMax returns (a) true if there is i such that $A[i] > x$, and (b) false if no such i exists.

```
function CHECKMAX($A, n, x$)
    for $i \leftarrow 0 \ldots n-1$ do
        if $A[i] > x$ then
            return True
        end if
    end for
    return False
end function
```

4. **The Horserace.** You are in charge of organizing a large horse race. There are 25 horses and you need to figure out the three fastest horses (*Win, Place,* and *Show*) by placing them into races. Unfortuantely, your track only has space for five horses, and you have no timing equipment. You're able to race five horses at a time and have decided to run multiple races to determine the three fastest horses. You can race any number of horses any number of times. However, you can only race five horses at a time, and you can't make direct comparison between results from two different races (since you don't keep track of exact times)
   (a) Give a method to determine the three fastest horses. What is the minimum number of races you need to arrange?
   (b) **optional extra credit.** same problem, but now you have n horses instead of 25. How many races do you need? Express your answer as a function of n. Do not use big-O notation.
   We'll award one extra credit point for the three best correct answers. You can assume $n = 5^k$ for some integer k.

5. Describe an algorithm for finding both the minimum and maximum (simultaneously) of n numbers using fewer than $3n/2$ *comparisons* of elements in A. The following pseudocode finds the max using $n - 1$ comparisons.

```
def findMax(Array A, size n>0):
  max=A[0]
  for i in 1 to n   (inclusive)
    if(A[i] > max)
      max=A[i]
  return max
```

Mystery Functions

In this part of the lab, you will first analyze six simple loop structures and determine their run time performance in terms of Big-O. Then, using the provided program `function_timer`, you will graph the empirical run times of these functions.

Functions

To begin, identify the Big-O run time for each of the following 6 functions. You do not need to provide a justification, simply write the function name followed by the Big-O on your solution. Use the strictest Big-O (i.e., the closest upper bound) and ignore all but the most significant term. (e.g. **Ex 7:** $O(n)$ **instead of** $O(n + 4\log n)$).

```
Ex1(n)
  for(i=0;  i<n;  i++){
    a=i;
  }

Ex2(n)
  for(i=0; i<n; i+=2){
    a=i;
  }

Ex3(n)
  for(i=0; i<n*n; i++){
    a=i;
  }

Ex4(n)
  for(i=0; i<n; i++){
    for(j=0; j<=i; j++){
        a=i;
    }
  }

Ex5(n)
  for(i=0; i<n*n; i++){
    for(j=0; j<=i; j++){
      a=i;
    }
  }

Ex6(n)
  k=1;
  for(i=0; i<n; i++){
    for(j=0; j<=k; j++){
      a=j;
    }
    k=k*2;
  }
```

Decoding functions using `function_timer`

Each of the above functions has implemented and packaged in the executable `function_timer`, which has been placed in your lab directory for this week. This program will provide an empirical run-time for each method and plots them automatically using the unix tool `gnuplot`.

To begin, here is the usage (this is an abbreviated version; obtain more details on the command line):

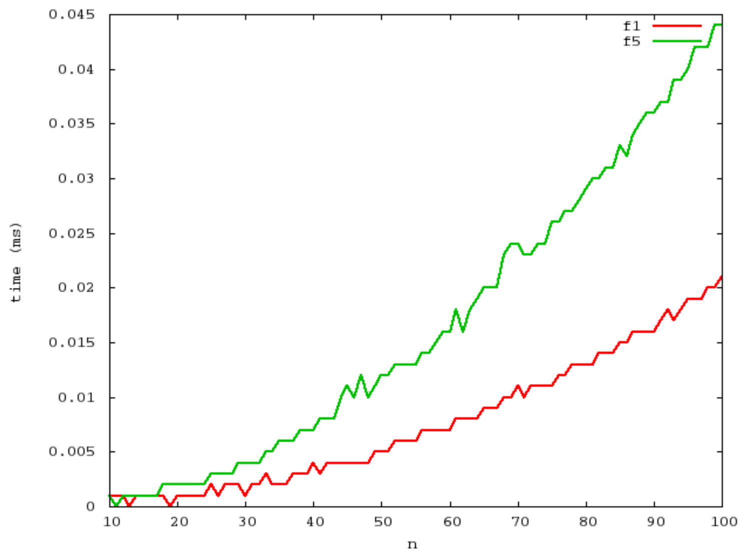
```
$ ./function_timer -h

OPTIONS:
         -h                print this help screen
         -n min_n          set the min value for n (dflt: 1)
         -m max_n          set the max value for n (dflt: 10)
         -[1-6]            Turn on mystery function number, e.g.
                           to run function 2 and 3: function_timer -2 -3
         -s out.png        Save the graph to a file out.png instead of graphing live
```

The key choices you have to make are:

- Which function(s) to plot. To plot `func1`, add `-1` as a command line argument. To plot `func2` vs `func3`, add `-2 -3`, etc.
- The minimum and maximum values for n. You should recall from lab that n may have to be very large for fast algorithms and small for slow ones. One size definitely does not fit all.
- Whether to save to file or immediately load plot. You'll want to save once you get a result you like by using the -s option.

For example, you can compare functions 1 and 5 from $n = 10 \ldots 100$ and view the result using gnuplot:

```
$ ./function_timer -1 -5 -n 10 -m 100 | gnuplot
```



Note that you must 'pipe' the output of the program to the plotting program, `gnuplot`. This is done using the vertical bar `|`, which sends the output from `function_timer` straight to `gnuplot`. To compare functions 1 and 5 from $n = 10 \ldots 100$ **and** save the output to a file named out1vs5.png, do the following:

```
$ ./function_timer -1 -5 -n 10 -m 100 -s out1vs5.png |gnuplot
```

Now here is the twist: I've jumbled up the ordering of functions in `function_timer`. Your job is to figure out the mapping from the functions in `function_timer` (e.g., `-1, -2, ...`) to their corresponding number above (e.g., `Ex1, Ex2, ...`). In addition to **determining the mapping between mystery functions and the functions above, you must support your argument in writing for each and provide a graph to support two of your mappings in your hard-copy submission.**